SMT Solvers, CBMC

http://d3s.mff.cuni.cz

Pavel Parízek

CHARLES UNIVERSITY IN PRAGUE
faculty of mathematics and physics
Z3

- SMT solver
  - theories: linear integer arithmetic, uninterpreted functions, arrays, bit vectors, ...
  - Input: SMT-LIB v2 (http://smtlib.cs.uiowa.edu/)

- Created by Microsoft Research

- Supports both Windows and Linux

- Source code & wiki: https://github.com/Z3Prover/z3

- Online interface: http://riseforfun.com/Z3
SMT-LIB: basics

; comment: after the semicolon until the end of a line

; specify the logic to be used
(set-logic QF_UFLIA)

; condition that should hold
(assert (= c (+ a 2))) ; c == a + 2
SMT-LIB: expressions

(not P)
(and b1 b2)
(or ...)
(xor ...)
(+ a b c d)
(= a b)
(=> true false)
SMT-LIB: predicates and functions

; function symbol
(declare-fun Plus (Int Int) Int)

; predicate
(declare-fun Odd (Int) Bool)

; constant
(declare-fun a () Int)
(declare-const c Int) ; syntactic sugar
SMT-LIB: example

(declare-const a Int)
(declare-const b Int)
(declare-const c Int)

(assert (<= a b))
(assert (<= b c))
(assert (<= c a))
(assert (= 6 (+ a b c)))

; result: sat, unsat, unknown
(check-sat)

(get-model)
Task 1: Triangle puzzle

- Goal
  - Fill the circles with different numbers from 1-9
  - Keep the sum of numbers on every side equal to 17
Task 1: Triangle puzzle

(declare-const c1 Int)
...

; TODO you must add constraints on c1 ... c9 to get the solution

; all different
(assert (not (= c1 c2))) (assert (not (= c1 c3))) ...
(assert (not (= c2 c3))) ...
...

; get results
(check-sat)
(get-model)
(declare-const c1 Int)
  ...

; sums for edges
(assert (= 17 (+ c1 c2 c3 c4)))
(assert (= 17 (+ c4 c5 c6 c7)))
(assert (= 17 (+ c7 c8 c9 c1)))

; in-range check
(assert (< 0 c1))
(assert (> 10 c1))
  ...

; all different
(assert (not (= c1 c2)))
(assert (not (= c1 c3))) ...
(assert (not (= c2 c3))) ...
  ...

; get results
(check-sat)
(get-model)
Kakuro puzzle

• **Goal**
  - Fill fields with numbers from 1-9
  - In a single row/column, the numbers may not repeat
  - Sums in the rows/columns must equal the specified number
Kakuro puzzle

23 30
16
17
35
7 11 10 16
21
6
24 17
29
15
7 8
7
12
12
16
3
int max(int a, int b) {
    int r;
    if (a < b) {
        r = b;
    }
    else { // a >= b
        r = a;
    }
    assert (a <= r && b <= r);
    return r;
}
int max(int a, int b) {
    int r;
    if (a < b) {
        r = b;
    }
    else { // a >= b
        r = a;
    }
    assert (a <= r && b <= r);
    return r;
}

a = *, b = *
r = *
if (a < b) {
    r = b;
}
else { // a >= b
    r = a;
}
assert (a <= r && b <= r);
How to encode programs using SMT

```
a = *, b = *
r = *
if (a < b) {
    r = b;
}
else { // a >= b
    r = a;
}
assert (a <= r && b <= r);
```

```
(declare-const a Int)
(declare-const b Int)
(declare-const r Int)

(assert (or
    (and (< a b) (= r b))
    (and (>= a b) (= r a)))
)

(assert (and (<= a r) (<= b r)))

(check-sat)
(get-model)
```
Variable assignments

```plaintext
int x = 0;
x = x + 1;
assert (x == 1);
```

```plaintext
(declare-const x1 Int)
(assert (= x1 0))
(assert (= x1 (+ x1 1)))
(assert (= x1 1))
(check-sat)
(get-model)
```
Variable assignments: versions & SSA

- Variables have multiple versions
- Static single assignment (SSA)

```plaintext
int x = 0;
x = x + 1;
assert (x == 1);
```

```plaintext
(declare-const x1 Int)
(declare-const x2 Int)

(assert (= x1 0))
(assert (= x2 (+ x1 1)))
(assert (= x2 1))
(check-sat)
(get-model)
```
Arrays

(declare-const c Int)

(declare-const a1 (Array Int Int))
(declare-const a2 (Array Int Int))

(assert (= c (select a1 10)))

(assert (= a2 (store a1 1 20)))
Bounded model checking

- Often used with SAT (propositional logic)
  - Q: Why?
  - Limited expressiveness: no “+” and “-”

- Q: How to encode statements like $x = a + b$?
Integer addition in HW/CPU

Image taken from Wikipedia
Encoding “Adder” as a SAT instance

\[ \begin{align*}
0x! & \quad ((A \land B \land C_{in}) \Rightarrow (S \land C_{out})) \land \\
1x! & \quad \begin{aligned}
((!A \land B \land C_{in}) \Rightarrow (!S \land C_{out})) & \land \\
((A \land !B \land C_{in}) \Rightarrow (!S \land C_{out})) & \land \\
((A \land B \land !C_{in}) \Rightarrow (!S \land C_{out})) & \land \\
((!A \land B \land !C_{in}) \Rightarrow (S \land !C_{out})) & \land \\
\end{aligned} \land \\
2x! & \quad \begin{aligned}
((!A \land B \land !C_{in}) \Rightarrow (S \land !C_{out})) & \land \\
((A \land !B \land !C_{in}) \Rightarrow (S \land !C_{out})) & \land \\
\end{aligned} \land \\
3x! & \quad ((!A \land !B \land !C_{in}) \Rightarrow (!!S \land !!C_{out})) \\
\end{align*} \]
Bounded model checker for program in C/C++

Developed at Oxford & Carnegie Mellon Uni

http://www.cprover.org/cbmc/

Source code and binaries freely available

Platforms: Windows, Linux, Mac OS
CBMC: how to use it

• Download
  - [http://www.cprover.org/cbmc/download/cbmc-5-4-win.zip](http://www.cprover.org/cbmc/download/cbmc-5-4-win.zip)

• Run from the Visual Studio Command Prompt
  - Why: correctly initialized environment

• Examples
  - [http://d3s.mff.cuni.cz/teaching/program_analysis_verification/files/bmc-examples.zip](http://d3s.mff.cuni.cz/teaching/program_analysis_verification/files/bmc-examples.zip)
Q: Exists an integer value \( x \) such that \( x \neq 0 \) and \( x == -x \) ?

Source code: `ex01-ints.c`
  - Q: Is the program safe or not?

Find the answer using CBMC
  - `cbmc64 bmc-examples\ex01-ints.c`
CBMC: example 2

- Program ex02-loops.c
  - Loop with bounded number of iterations

- Command line argument “-function”
  - Specifies an entry point

- Usage
  - cbmc64 ex02-loops.c -function sum
CBMC: example 3

- Program ex03-fact.c
  - Unbounded loop
  - Infinite unwinding

- Command line argument “--unwind N”

- Argument “--unwinding-assertions”
• Program ex04-binsearch.c
  ▪ Loop bound cannot be determined statically

• Supported built-in properties
  ▪ Checking bounds for accesses to array elements
    ▪ Parameter “--bounds-check”
  ▪ Checking null dereferences
    ▪ Parameter “--pointer-check”