LINEAR TEMPORAL LOGIC (LTL)

- Captures properties of particular runs – executions
- Does not capture possible futures – branching
- Frequently used for expressing system properties
LTL syntax defined inductively, similarly to propositional logic:

Let \( \text{AP} \) be a finite set of Boolean variables (atomic propositions). The set of LTL formulae over \( \text{AP} \) is defined as:

- If \( p \in \text{AP} \) then \( p \) is LTL formula.
- If \( \varphi \) and \( \psi \) are LTL formulae then \( \neg \varphi, \varphi \lor \psi, \varphi \land \psi, X \varphi, \varphi U \psi, F \varphi, \varphi R \psi, \) and \( G \varphi \) are LTL formulae.

Negation, disjunction, \( X \), and \( U \) are fundamental operators, others can be derived.
Path in Kripke structure is infinite sequence $\pi = \pi_0, \pi_1, \pi_2, \ldots$ where for all $\forall i > 0. (\pi_i, \pi_{i+1}) \in R$

Let $M = (S, I, R, L)$ be Kripke structure and $\pi = \pi_0, \pi_1, \pi_2, \ldots$ be an infinite path in $M$. For an integer $i \geq 0$, $\pi^i$ stands for i-th suffix of $\pi$: $\pi^i = \pi_i, \pi_{i+1}, \pi_{i+2}, \ldots$

Let $M$ be Kripke structure, $\varphi$ be LTL formula, $\pi$ be path in $M$ and $s$ be state of $M$.

$M, \pi \models \varphi$: Path $\pi$ from $M$ satisfies $\varphi$

$M, s \models \varphi$: State $s$ from $M$ satisfies $\varphi$

- $M, s \models \varphi \iff \forall \pi. \pi_0 = s : M, \pi \models \varphi$
LTL SEMANTICS

\[ M, \pi \models p \iff p \in L(\pi_0) \]
\[ M, \pi \models \neg \varphi \iff \neg (M, \pi \models \varphi) \]
\[ M, \pi \models \varphi_1 \lor \varphi_2 \iff M, \pi \models \varphi_1 \lor M, \pi \models \varphi_2 \]
\[ M, \pi \models \varphi_1 \land \varphi_2 \iff M, \pi \models \varphi_1 \land M, \pi \models \varphi_2 \]
\[ M, \pi \models X \varphi \iff M, \pi^1 \models \varphi \]
\[ M, \pi \models F \varphi \iff \exists i \geq 0. M, \pi^i \models \varphi \]
\[ M, \pi \models G \varphi \iff \forall i \geq 0. M, \pi^i \models \varphi \]
\[ M, \pi \models \varphi_1 U \varphi_2 \iff \exists i \geq 0. M, \pi^i \models \varphi_2 \land \forall j. 0 \leq j < i \implies M, \pi^j \models \varphi_1 \]
\[ M, \pi \models \varphi_1 R \varphi_2 \iff (G \varphi_2) \lor (\varphi_2 U (\varphi_1 \land \varphi_2)) \]
Decide and prove or disprove equivalence of following pairs of LTL formulae:

- $Gp : FGp$
- $Gp : GFp$
- $Fp : FGp$
- $Fp : GFp$
- $p U q : Gq \lor (Fq \land Gp)$

Is any of these formulae implied by another one?
Assume model of Alternating Bit Protocol (last lab)
What properties could be verified?
Express them in LTL!
Assume following model of microwave oven:

What properties could be verified?
Express them in LTL.
Would you enhance the model somehow?