Syllabus I.

- Mathematical structures for behaviour modelling:
  - Labelled transition systems, Kripke structures, Timed automata, Markov chains
- Specification of system properties
  - Temporal logics: LTL, CTL, TCTL, PCTL
- Basic verification tasks
  - Equivalence checking and model checking
Decidability and complexity
  ... of equivalence checking and model checking with respect to model type
Software tools for model checking
Hard issues in formal verification
  Infinite-state systems
  State explosion problem
  Strategies to fight it
Final grades will be determined by the quality of homework and the result of the final exam in the following ratio:

- 55% Assignments (several home works)
- 45% Final exam (max. 100 points)
  - >= 80 → 1
  - >= 71 → 2
  - >= 62 → 3

Final exam—written test
All information available at the course web page:

https://d3s.mff.cuni.cz/teaching/nswi101/
Part I: Introduction
Need for formal methods: striking stories

Ariane 5, 1996

- False angle of attack caused by incorrect altitude data following software exception
- The rocket self-destructed in 37 seconds after launch
- Software exception—overflow in conversion of 64-bit floating-point number to 16-bit signed integer value caused operand error
- The error occurred because of unexpectedly high value of sensed horizontal velocity
- The value was much higher than expected because early part of Ariane 5 trajectory differed from that of Ariane 4—higher velocity values
- Direct cost €500M, indirect cost €2,000M
NEED FOR FORMAL METHODS: STRIKING STORIES

Intel: Pentium FDIV bug, 1994

- “Imprecision” of FDIV operation firstly not admitted
- CPUs called off after publishing proof
- Cost $500M
NEED FOR FORMAL METHODS: STRIKING STORIES

NASA Mars Climate Orbiter
- September 30, 1999
- Peer review preliminary findings indicate that one team used English units while the other used metric units for key spacecraft operations

NASA Mars Polar Lander
- December, 1999
- The leading theory is that surface contact detector located on landing struts mistakenly interpreted the force of landing struts deployment as contact with the surface, causing landing rockets to shut down prematurely and probe to impact at a too-high velocity
NEED FOR FORMAL METHODS: STRIKING STORIES

Nissan, 2015

- Sensor failure caused not detecting human in the seat
- Airbag malfunction (failing in car crash) appeared
- In 2015, Nissan recalled 3.5 millions of cars to fix this
- Airbag problems reported by other car manufacturers in 2016: General Motors—GMC, Chevrolet, Buick, Cadillac
NEED FOR FORMAL METHODS: STRIKING STORIES

Boeing 737 MAX, 2018

- New device introduced into Boeing 737 MAX (MCAS) to compensate too steep take-off
- MCAS relies on single sensor
- MCAS can reset itself after a pilot intervention
- Info on MCAS was not put into manuals!
- Two air crafts with passengers crashed (2018 and 2019)
- The air crafts were grounded
ACHIEVING SYSTEM RELIABILITY

Experimental methods
- Testing—applied to the system itself
- Simulation—experimenting with a model of a system
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Formal methods
- Deductive verification—theorem proving
- Equivalence checking—comparing two specifications (models)
- Model checking—checking a particular property of a model (even code)
Program execution vs. Verification

Program execution

\[
\begin{align*}
S_0 & \rightarrow \text{coin} \rightarrow S_1 \\
S_1 & \rightarrow \text{coffee} \rightarrow S_2 \\
S_2 & \\
S_0 & \rightarrow \text{coin} \rightarrow S_3 \\
S_3 & \rightarrow \text{tea} \rightarrow S_4
\end{align*}
\]

Program verification

\[
\begin{align*}
S_0 & \rightarrow \text{coin} \rightarrow S_1 \\
S_1 & \rightarrow \text{coffee} \rightarrow S_2 \\
S_2 & \\
S_0 & \rightarrow \text{coin} \rightarrow S_3 \\
S_3 & \rightarrow \text{tea} \rightarrow S_4
\end{align*}
\]
Part II: Labelled Transition Systems
\begin{align*}
x &:= 0; \\
y &:= 0; \\
\text{for } i &:= 1 \text{ to } 3 \{ \\
& \quad x := x + 1; \\
& \quad y := y + 1; \\
\}
\end{align*}
Labelled Transition System is a triple \((S, \text{Act} \rightarrow)\):  
- \(S\) is set of states (domain) 
- \(\text{Act}\) is set of labels (actions) 
- \(\rightarrow\) is transition relation: \(\rightarrow \subseteq S \times \text{Act} \times S\)
LTS VS. FINITE AUTOMATON

LTS:
- \((S, \text{Act}, \rightarrow)\)
- Trace—sequence of labels following one path in LTS
- LTS corresponds to set of traces reachable in the LTS

Finite Automaton:
- \((S, \text{Act}, \rightarrow, I, A)\)
- Additionally sets of initial and accepting states
- Notion of word and language accepted by automaton
traces(s) = \{ \sigma \in Act^* | s \xrightarrow{\sigma} \}
traces(s_0) = \{ \epsilon, \text{coin}, \text{coin.coffee}, \text{coin.tea} \}

Note that due to absence of initial and accepting states, trace can terminate at any state.
COMPARING LTSSs

Many relations to compare LTSSs between each other:
- Trace preorder/equivalence
- Simulation preorder/equivalence
- Bisimilarity
- Readiness equivalence
- Failure equivalence
- ...

- strong bisimilarity
- 2-nested simulation equivalence
- ready simulation equivalence
- possible-futures equivalence
- ready trace equivalence
- readiness equivalence
- failure trace equivalence
- simulation equivalence
- failure equivalence
- completed trace equivalence
- trace equivalence
States $s$ and $t$ are in trace preorder relation ($s \leq_t t$) iff
\[ \text{traces}(s) \subseteq \text{traces}(t) \]

States $s$ and $t$ are trace equivalent ($s =_t t$) iff
\[ \text{traces}(s) \subseteq \text{traces}(t) \wedge \text{traces}(t) \subseteq \text{traces}(s) \]

This corresponds to equivalence of languages in the automata world.
Relation $R \subseteq S \times S$ is **simulation** iff $(s, t) \in R \implies \forall s'. s \xrightarrow{a} s' \exists t'. t \xrightarrow{a} t' \land (s', t') \in R$

States $s$ and $t$ are in **simulation preorder** $(s \leq_s t)$ iff there exists simulation $R$ and $(s, t) \in R$.

States $s$ and $t$ are **equivalent under simulation** $(s =_s t)$ iff

$$\exists R, Q. (s, t) \in R \land (t, s) \in Q, \text{ and } R \text{ and } Q \text{ are simulations.}$$

*R and Q can be the same relation or not.*
LTS relations—Examples

Trace equivalence:

\[
\text{traces}(s_0) = \text{traces}(s_1) = \text{traces}(s_2) = \{\epsilon, \text{coin}, \text{coin.coffee}, \text{coin.tea}\}
\]

\[s_0 \models t \quad s_1 \models t \quad s_2\]
Simulation preorder and equivalence:

\[ s_0 \leq s_1 \land \neg (s_1 \leq s_0) \implies s_0 \neq s_1 \]

\[ s_1 \leq s_2 \land s_2 \leq s_1 \implies s_1 = s_2 \]
Relation $R$ is **bisimulation** iff $\forall s, t. (s, t) \in R \implies$

$$\forall s'. s \xrightarrow{a} s' \exists t'. t \xrightarrow{a} t' \land (s', t') \in R \land$$

$$\forall t'. t \xrightarrow{a} t' \exists s'. s \xrightarrow{a} s' \land (s', t') \in R$$

States $s$ and $t$ are **bisimilar** (equivalent under bisimulation) $(s \sim t)$ iff

$(s, t) \in R$ and $R$ is bisimulation.
BISIMILARITY—EXAMPLE

\[
\begin{align*}
    s_0 \sim & s_1 \\
    s_1 \sim & s_2 \\
    s_0 \sim & s_2
\end{align*}
\]

Jan Kofroň: Behaviour Models and Verification
Bisimilarity—Example

\[ S_0 \sim S_1 \]

Diagram:

- \( S_0 \)
  - coin
  - tea
  - \( S_0 \)
- \( S_1 \)
  - coin
  - tea
  - \( S_1 \)
For deterministic LTS, all the relations are equivalent: $a \equiv_t b \iff a \equiv_s b \iff a \sim b$.

Non-deterministic LTS cannot be transformed into equivalent deterministic LTS as in automata world.

- It can, of course, but its semantics changes!
**PROCESS ALGEBRAS (PA)**

- Textual way for capturing LTS
- Various process algebras exist: CCS, CSP, ACP, $\pi$-calculus, $\mu$-calculus, ...
- Equational reasoning—transformations of expressions usually to simplify them or to proof certain property
- Modelling in many areas: concurrent systems, communication protocols, electronic circuits, biochemical processes, ...
Simple process algebra by Jan Bergstra and Jan Willem Klop (1982)

Just few syntactical constructs:
- Choice (+)
- Sequencing (.)
- Concurrency (||)
- Process communication (γ)
- Abstraction (τ)

Example of processes:
- p : (gen₁ + gen₂).send
- q : recv.proc
- Defining communication: γ(send, recv) = trans
- Composition of processes: p||q = (gen₁ + gen₂).trans.proc
- Hiding internal computation (abstraction): τ{gen₁,gen₂,proc}(p||q) = τ.trans.τ
ACP SEMANTICS

For process variables $x, y$

- $x + y = y + x$
- $(x + y) + z = x + (y + z)$
- $x + x = x$
- $(x + y).z = x.z + y.z$
- $(x.y).z = x.(y.z)$
- $x + \delta = x$
- $\delta.x = \delta$

Note that $z.(x + y) = z.x + z.y$ is not included (non-deterministic choice)!
Let $A = a.b.\epsilon$ and $B = b.c.\epsilon$

Parallel composition just “syntax sugar”:

$A || B = a.(b.b.c.\epsilon + b.(b.c.\epsilon + c.b.\epsilon)) + b.(a.(b.c.\epsilon + c.b.\epsilon) + c.a.b.\epsilon)$
Let $A = a.b.\epsilon$ and $B = b.c.\epsilon$

Let $\gamma(b, b) = d$

Processes can perform the actions synchronously.
PARALLEL COMPOSITION IN ACP WITH ENFORCED COMMUNICATION

Let $A = a.b.\epsilon$ and $B = b.c.\epsilon$
Let $\gamma(b, b) = d$

Disabling (encapsulation) operator:
$\delta\{b\}(A||B) = a.d.c.\epsilon$

Processes must perform actions synchronously.
BUILDING CONCURRENT SYSTEMS

1. Specify particular components
2. Specify communication actions
3. Construct parallel compositions
4. Disable certain actions to enforce communication
LTS relations useful for verifying design of communication protocols, cryptography protocols, and algorithms in general. They are also applicable for checking correspondence between code (implementation) and LTS (specification).

- Inherently hard (undecidable) problem—models made of finite number of states while code usually induces infinite state space.
- Preorder relation usually applied—specification to be implemented (can implement more).
Part III: Thesis topics
If you are interested in model checking (or verification in general), contact me for supervising bachelor or master thesis, software project, research project, or just contributing to one of our verification projects!

For a (non-exhaustive) topic list, please visit

https://d3s.mff.cuni.cz/students/topics/

Surely we can devise a topic suiting your expectations!