SMT Solvers, CBMC

http://d3s.mff.cuni.cz

Department of Distributed and Dependable Systems

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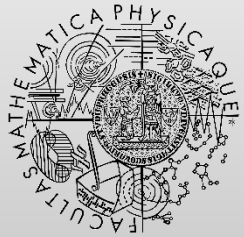

FACULTY OF MATHEMATICS AND PHYSICS
Charles University

Z3

- SMT solver
  - theories: linear integer arithmetic, uninterpreted functions, arrays, bit vectors, ...
  - Input: SMT-LIB v2 (http://smtlib.cs.uiowa.edu/)
- Created by Microsoft Research
- Supports both Windows and Linux
- Source code & wiki: https://github.com/Z3Prover/z3
- Online interface: https://jfmc.github.io/z3-play/

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SMT-LIB: basics

```
; comment: after the semicolon until the end of a line
;

; specify the logic to be used
(set-logic QF_UFLIA)

; condition that should hold
(assert (= c (+ a 2)))          ; c == a + 2
```

SMT-LIB: expressions

```
(not P)
(and b1 b2)
(or ...)
(xor ...)

(+ a b c d)
(= a b)
(=> true false)
```

SMT-LIB: predicates and functions

```
; function symbol
(declare-fun Plus (Int Int) Int)

; predicate
(declare-fun Odd (Int) Bool)

; constant
(declare-fun a () Int)
(declare-const c Int)  ; syntactic sugar
```

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SMT-LIB: example

```
(declare-const a Int)
(declare-const b Int)
(declare-const c Int)

(assert (<= a b))
(assert (<= b c))
(assert (<= c a))
(assert (= 6 (+ a b c)))

; result: sat, unsat, unknown
(check-sat)

(get-model)
```

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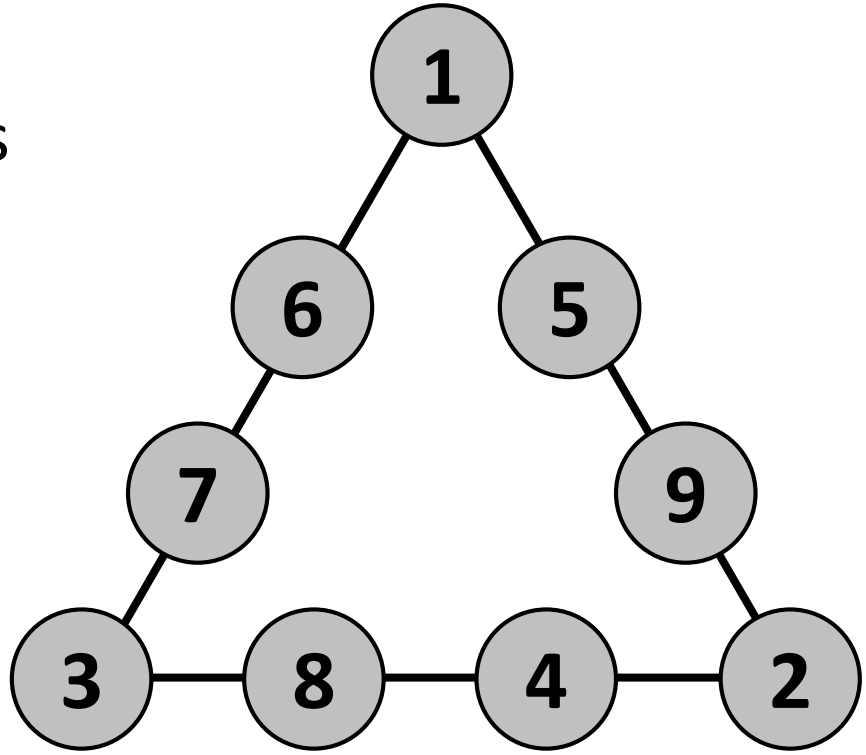
Task 1: Triangle puzzle

- Goal
  - Fill the circles with different numbers from 1-9
  - Keep the sum of numbers on every side equal to 17



Task 1: Triangle puzzle

```
(declare-const c1 Int)
 ...

; TODO you must add constraints on c1 ... c9 to get the solution




; all different
(assert (not (= c1 c2))) (assert (not (= c1 c3))) ...
(assert (not (= c2 c3))) ...
 ...

; get results
(check-sat)
(get-model)
```

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Task 1: Triangle puzzle

```
(declare-const c1 Int)
 ...

; sums for edges
(assert (= 17 (+ c1 c2 c3 c4)))
(assert (= 17 (+ c4 c5 c6 c7)))
(assert (= 17 (+ c7 c8 c9 c1)))

; in-range check
(assert (< 0 c1))
(assert (> 10 c1))
 ...

; all different
(assert (not (= c1 c2))) (assert (not (= c1 c3))) ...
(assert (not (= c2 c3))) ...
 ...

; get results
(check-sat)
(get-model)
```

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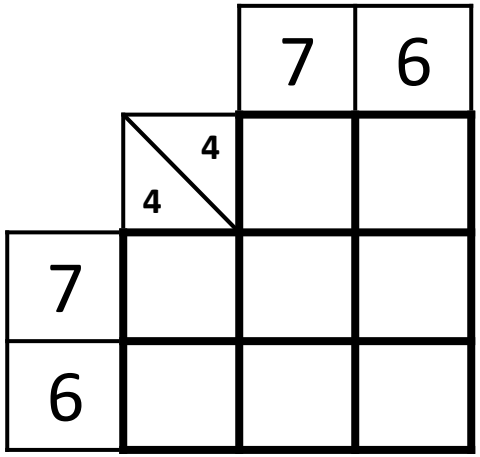
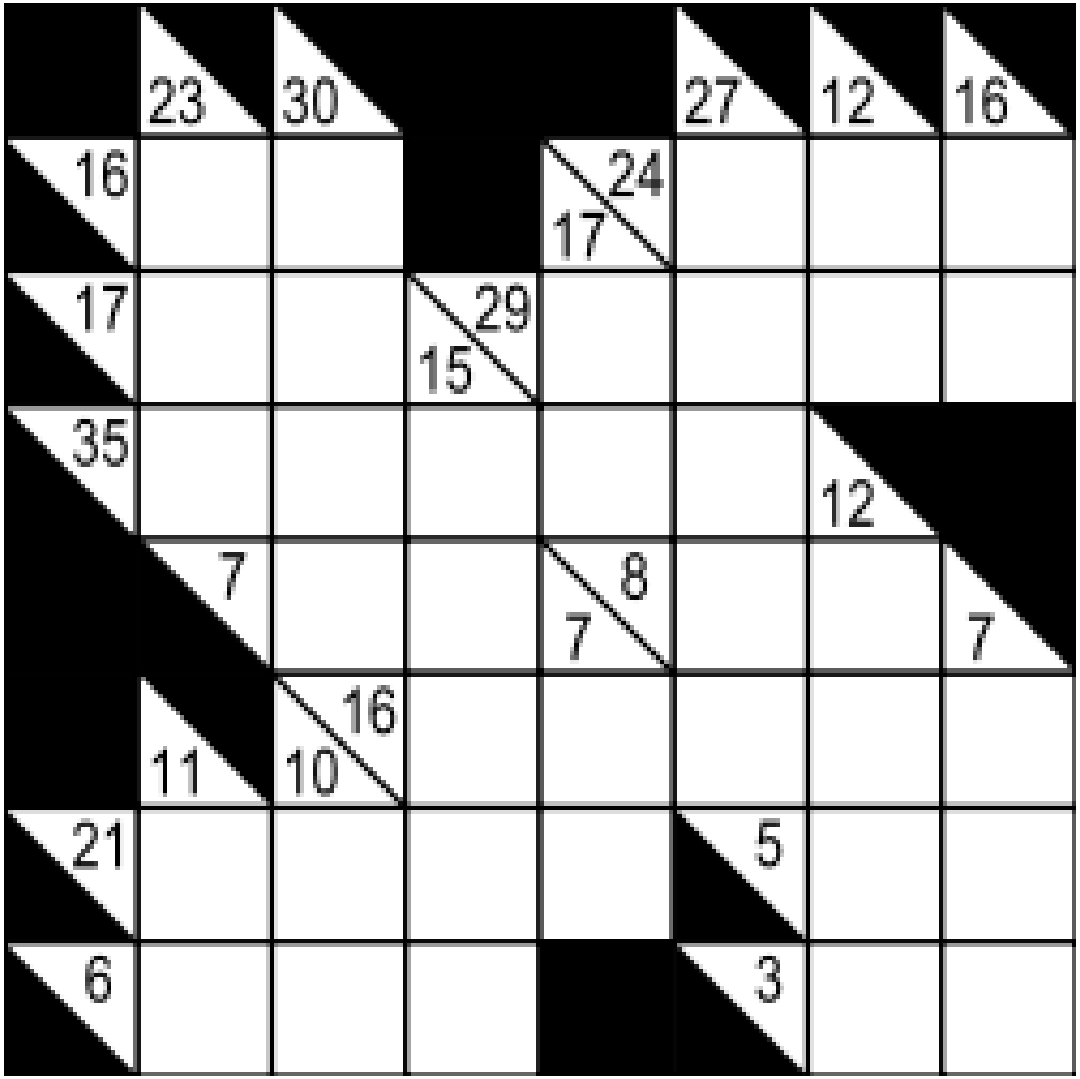
Kakuro puzzle

- Goal
  - Fill fields with numbers from 1-9
  - In a single row/column, the numbers may not repeat
  - Sums in the rows/columns must equal the specified number



Kakuro puzzle



How to encode programs using SMT

```
int max(int a, int b) {
  int r;
  if (a < b) {
    r = b;
  }
  else { // a >= b
    r = a;
  }
  assert (a <= r && b <= r);
  return r;
}
```

How to encode programs using SMT

```
int max(int a, int b) {
  int r;
  if (a < b) {
    r = b;
  }
  else { // a >= b
    r = a;
  }
  assert (a <= r && b <= r);
  return r;
}
```

```
a = *, b = *
r = *
if (a < b) {
  r = b;
}
else { // a >= b
  r = a;
}

assert (a <= r && b <= r);
```

How to encode programs using SMT

```
a = *, b = *
r = *
if (a < b) {
  r = b;
}
else { // a >= b
  r = a;
}

assert (a <= r && b <= r);
```

```
(declare-const a Int)
(declare-const b Int)
(declare-const r Int)

(assert (or
  (and (< a b) (= r b))
  (and (>= a b) (= r a))
))

(assert (and (<= a r) (<= b r)))

(check-sat)
(get-model)
```

Variable assignments

```
int x = 0;

x = x + 1;

assert (x == 1);
```

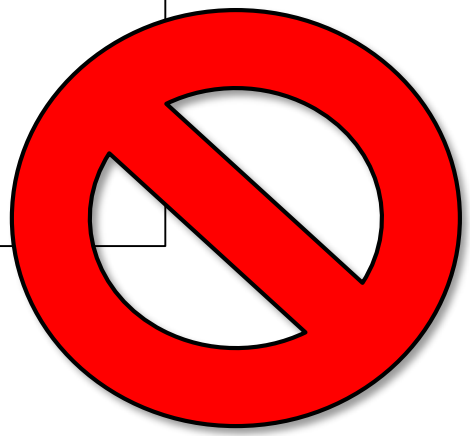
```
(declare-const x1 Int)

(assert (= x1 0))

(assert (= x1 (+ x1 1)))

(assert (= x1 1))

(check-sat)
(get-model)
```



Variable assignments: versions & SSA

- Variables have multiple versions
- Static single assignment (SSA)

```
int x = 0;

x = x + 1;

assert (x == 1);
```

```
(declare-const x1 Int)
(declare-const x2 Int)

(assert (= x1 0))

(assert (= x2 (+ x1 1)))

(assert (= x2 1))

(check-sat)
(get-model)
```

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Arrays

```
(declare-const c Int)

(declare-const a1 (Array Int Int))
(declare-const a2 (Array Int Int))

(assert (= c (select a1 10)))

(assert (= a2 (store a1 1 20)))
```

Bounded model checking

- Often used with SAT (propositional logic)
  - **Q: Why?**
  - Limited expressiveness: no “+” and “-”
  - **Q: How to encode statements like x = a + b ?**

Integer addition in HW/CPU

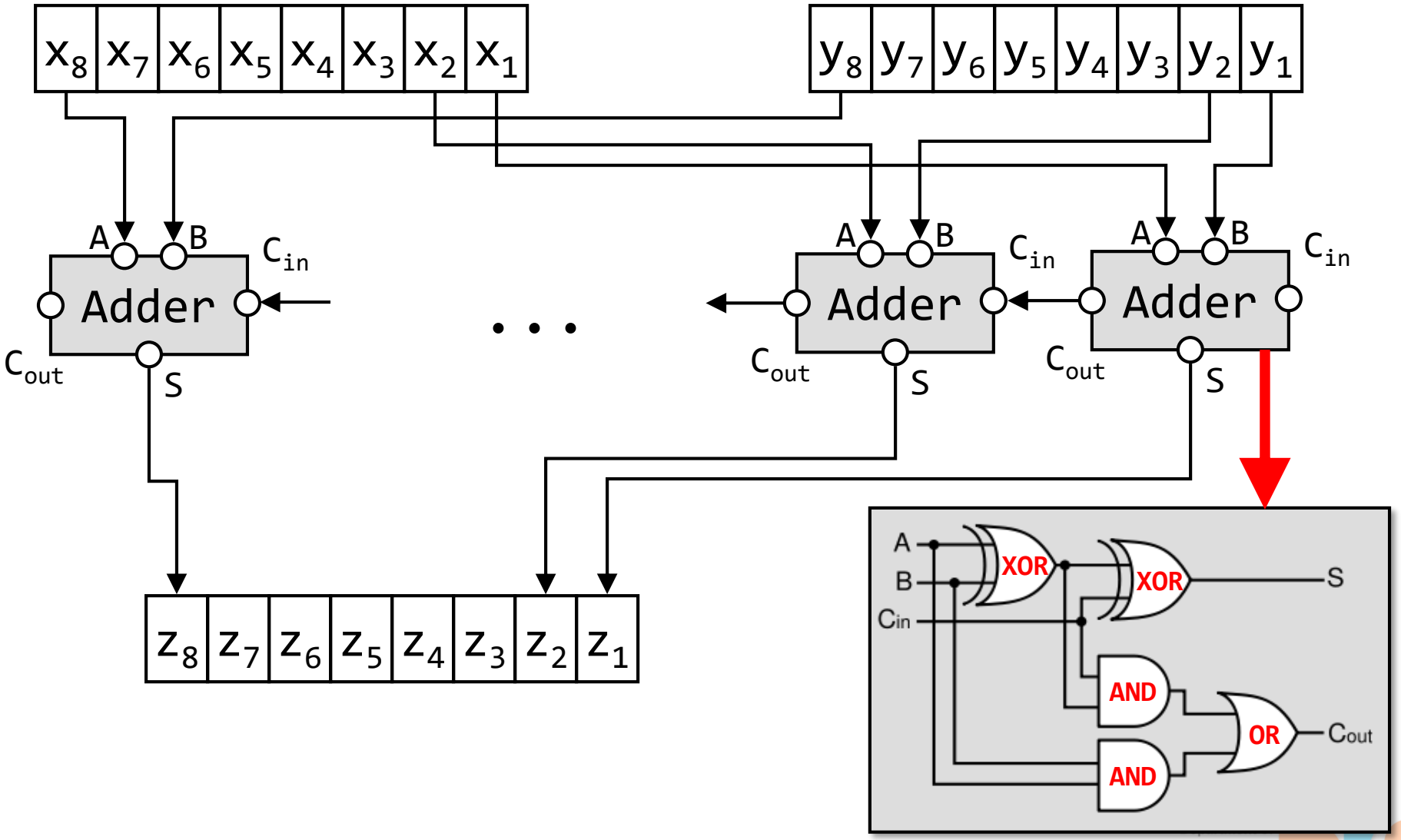


Image taken from Wikipedia

Encoding “Adder” as a SAT instance

0x! $((A \& B \& C_{in}) \Rightarrow (S \& C_{out})) \&$

1x! $((\neg A \& B \& C_{in}) \Rightarrow (\neg S \& C_{out})) \&$

1x! $((A \& \neg B \& C_{in}) \Rightarrow (\neg S \& C_{out})) \&$

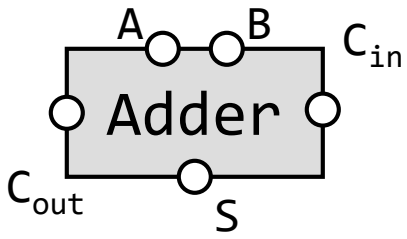
1x! $((A \& B \& \neg C_{in}) \Rightarrow (\neg S \& C_{out})) \&$

2x! $((\neg A \& \neg B \& C_{in}) \Rightarrow (S \& \neg C_{out})) \&$

2x! $((\neg A \& B \& \neg C_{in}) \Rightarrow (S \& \neg C_{out})) \&$

2x! $((A \& \neg B \& \neg C_{in}) \Rightarrow (S \& \neg C_{out})) \&$

3x! $((\neg A \& \neg B \& \neg C_{in}) \Rightarrow (\neg S \& \neg C_{out}))$



CBMC

- Bounded model checker for program in C/C++
- Developed at Oxford & Carnegie Mellon Uni
- http://www.cprover.org/cbmc/
- Source code and binaries freely available
  - Platforms: Windows, Linux, Mac OS

CBMC: how to use it

- Download
  - http://www.cprover.org/cbmc/download/cbmc-5-4-win.zip
- Run from the Visual Studio Command Prompt
  - Why: correctly initialized environment
- Examples
  - http://d3s.mff.cuni.cz/files/teaching/nswi132/files/bmc-examples.zip

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CBMC: example 1

- Q: Exists an integer value *x* such that *x* != 0 and *x* == -*x* ?

- Source code: `ex01-ints.c`
  - Q: Is the program safe or not ?

- Find the answer using CBMC
  - `cbmc64 bmc-examples\ex01-ints.c`

CBMC: example 2

- Program `ex02-loops.c`
  - Loop with bounded number of iterations
- Command line argument "`-function`"
  - Specifies an entry point
- Usage
  - `cbmc64 ex02-loops.c -function sum`

CBMC: example 3

- Program `ex03-fact.c`
  - Unbounded loop
  - Infinite unwinding
- Command line argument "`--unwind N`"
- Argument "`--unwinding-assertions`"

CBMC: example 4

- Program `ex04-binsearch.c`
  - Loop bound cannot be determined statically
- Supported built-in properties
  - Checking bounds for accesses to array elements
    - Parameter "`--bounds-check`"
  - Checking null dereferences
    - Parameter "`--pointer-check`"

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