Petri Nets
Petri nets

- Modeling language
  - concurrent and distributed SW systems
  - reactive systems (asynchronous events)

- Notations: graphical, mathematical

- Many variants and extensions
  - Basic (ordinary)
  - Colored (CPN)
  - Hierarchical nets
Basic elements

- Places
- Transitions
- Arcs
- Tokens
Semantics

• Marking
  - Function $M : P \rightarrow N$

• Transitions
  - Enabled: when input places contain enough tokens
  - Firing (execution)
    - Removing tokens from input places
    - Adding tokens to output places
Examples

- Conflicting transitions
- Independent transitions
- Synchronization
Definition

Petri net is a tuple \((P, T, A, w, M_0)\), where:

- \(A \subseteq (P \times T) \cup (T \times P)\)
- \(P \cap T = \emptyset\) (disjunct)
- \(w: A \rightarrow \mathbb{N}\) is a weight function
- \(M_0: P \rightarrow \mathbb{N}\) is the initial marking

Reachability graph \(R\)

- \(M_0 \in R\)
- \(M \in R \land t \in T\) enabled in \(M\) s.t. \(M \xrightarrow{t} M' \Rightarrow M' \in R\)
Example: dining philosophers

- Two philosophers
- Two shared forks
Properties

- Reachability of M
  - ∃ sequence of transitions from $M_0$ to M
- Reachable markings $R(M)$

- Coverability of M
  - ∃ $M' \in R(M_0)$ such that $\forall p \cdot M'(p) \geq M(p)$

- Applications: verification, simulation, analysis
Variants

- **Ordinary Petri net**
  - every arc has the weight 1

- **State machine**
  - every transition has exactly one input place and one output place

- **Colored Petri Nets**
Colored Petri Nets (CPN)

- Support for data types and manipulation

- Multiple types of tokens (colors)
  - data type = set of values \( \approx \) set of colors
  - token value \( \approx \) token color

- New elements
  - Places: color sets (allowed token types)
  - Transitions: guard conditions (enabling)
  - Arcs: arc expressions (transferring values)
CPN: usage

- Example
  - Distributed storage system with a very simple protocol for synchronization
    - Entities: client, server, data storage

- Applications
  - Communication protocols
  - Distributed algorithms
  - Control for embedded systems
Tools

- Popular editors for creating diagrams
  - [https://app.diagrams.net/](https://app.diagrams.net/) (former draw.io)

- PetriDotNet

- CPN Tools
  - [http://cpntools.org/](http://cpntools.org/)
  - [http://cpntools.org/download](http://cpntools.org/download)

- PIPE 2
Literature

- Basic Petri Nets
  - Further details and references to various literature

- Colored Petri Nets