Petri nets

- Modeling language
  - concurrent and distributed SW systems
  - reactive systems (asynchronous events)

- Notations: graphical, mathematical

- Many variants and extensions
  - Basic (ordinary)
  - Colored (CPN)
  - Hierarchical nets
Basic elements

- Places
- Transitions
- Arcs
- Tokens
Semantics

- **Marking**
  - Function \( M : P \rightarrow N \)

- **Transitions**
  - Enabled: when input places contain enough tokens
  - Firing (execution)
    - Removing tokens from input places
    - Adding tokens to output places
Examples

- Conflicting transitions
- Independent transitions
- Synchronization
**Definition**

Petri net is a tuple \((P, T, A, w, M_0)\), where:

- \(A \subseteq (P \times T) \cup (T \times P)\)
- \(P \cap T = \emptyset\) (disjunct)
- \(w: A \rightarrow N\) is a weight function
- \(M_0: P \rightarrow N\) is the initial marking

Reachability graph \(R\)

- \(M_0 \in R\)
- \(M \in R \land t \in T\) enabled in \(M\) s.t. \(M \xrightarrow{t} M' \Rightarrow M' \in R\)
Example: dining philosophers

- Two philosophers
- Two shared forks
Properties

• Reachability of $M$
  - $\exists$ sequence of transitions from $M_0$ to $M$

• Reachable markings $R(M)$

• Coverability of $M$
  - $\exists M' \in R(M_0)$ such that $\forall p \cdot M'(p) \geq M(p)$

• Applications: verification, simulation, analysis
Variants

• Ordinary Petri net
  ▪ every arc has the weight 1

• State machine
  ▪ every transition has exactly one input place and one output place

• Colored Petri Nets
Colored Petri Nets (CPN)

- Support for data types and manipulation
- Multiple types of tokens (colors)
  - data type = set of values ≈ set of colors
  - token value ≈ token color
- New elements
  - Places: color sets (allowed token types)
  - Transitions: guard conditions (enabling)
  - Arcs: arc expressions (transferring values)
Example
- Distributed storage system with a very simple protocol for synchronization
  - Entities: client, server, data storage

Applications
- Communication protocols
- Distributed algorithms
- Control for embedded systems
• Popular editors for creating diagrams
  ▪ https://app.diagrams.net/ (former draw.io)

• PetriDotNet

• CPN Tools
  ▪ http://cpntools.org/
  ▪ http://cpntools.org/download

• PIPE 2
  ▪ http://pipe2.sourceforge.net/
• Basic Petri Nets
    • Further details and references to various literature

• Colored Petri Nets