Invariants production and Transition Power Abstraction

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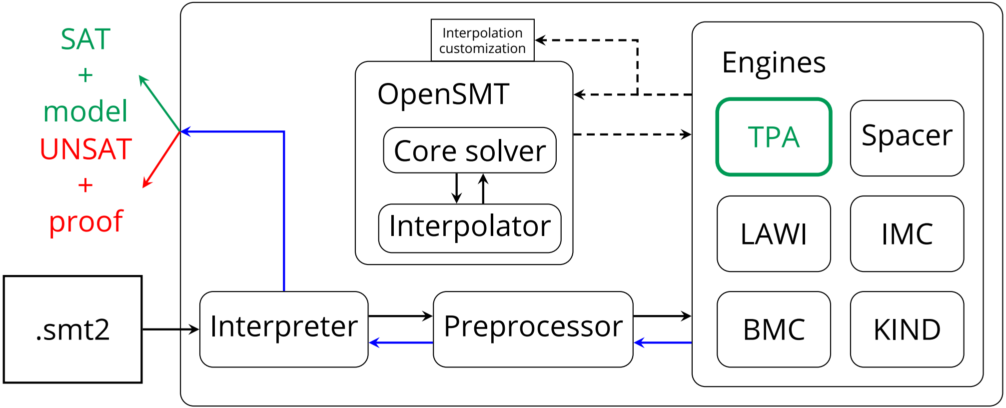
Golem Architecture[1]



[1] Blicha, Britikov, and Sharygina, "The Golem Horn Solver", *Computer Aided Verification - 35th International Conference, CAV 2023, Paris, France, July 17-22, 2023, Proceedings, Part II*, 2023.

Motivation for Transition Power Abstraction (TPA)[4]

- All of the model checking engines like Spacer,[2] LAWI[3] are concentrated on states.
- Classical engines are slow in some cases (for example for deep loops).

- TPA abstracts over transitions.
- TPA goes deep, finding complicated counterexamples.
- TPA turned out to be able to prove safety.

[2] Komuravelli, Gurfinkel, and Chaki, "SMT-based Model Checking For Recursive Programs", *FMSD*, 2016.
[3] McMillan, "Lazy Abstraction with Interpolants", *CAV*, 2006.
[4] Blicha et al., "Transition Power Abstractions for Deep Counterexample Detection", *TACAS*, 2022.

Transition Power Abstraction

$$\boxed{\mathrm{Init}(x^0)} \longrightarrow \boxed{Tr(x^0, x^1)} \longrightarrow \boxed{\mathrm{Bad}(x^1)}$$

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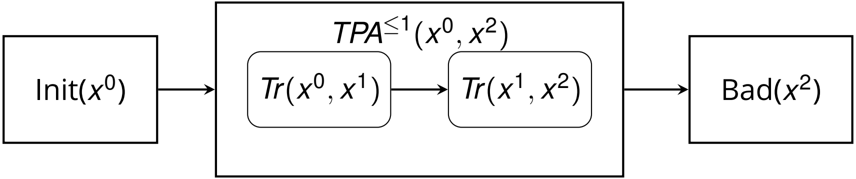

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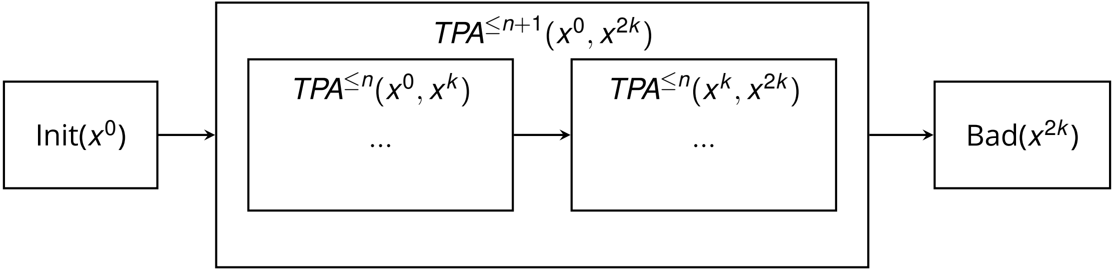

Where $k = 2^n$.

Transition Power Abstraction (TPA)

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Transition Power Abstraction (TPA) algorithm is based on abstract transition sequence $TPA^{\leq 0}, TPA^{\leq 1}, \ldots, TPA^{\leq n}, \ldots$

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- Quantifier-free (only 2 copies of state variables)
- Construction and refinement of the sequence intertwined with bounded reachability checks

Split Transition Power Abstraction (split-TPA)[5]

Concept

Adds additional checks to the TPA algorithm, now the reachability is split on two types of abstract transitions:

- $TPA^{<n+1} = TPA^{<n} \cup TPA^{=n} \circ TPA^{<n}$
- $TPA^{=n+1} = TPA^{=n} \circ TPA^{=n}$

This approach has folloving positive effects:

- Smaller, simpler checks.

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- More invariant candidates.

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Safety checks in TPA

Both TPA and split-TPA support the production of the safety invariants. Following conditions should be satisfied for $TPA^{<n}$ to be safe inductive transition invariant:

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- $TPA^{<n}(x, x') \wedge Tr(x', x'') \implies TPA^{<n}(x, x'')$.
- $Init(x) \wedge TPA^{<n}(x, x') \wedge Bad(x') \implies false$.

Houdini search[6]

Houdini search - is a general algorithm to find the biggest inductive subset in a formula. Was originaly introduced to search for loop invariants.

1. We have a set of invariant candidates $C(x) = c_1(x) \wedge c_2(x) \wedge ... \wedge c_n(x)$, and some kind of transition $Tr(x, x')$.

[6] Flanagan and Leino, "Houdini, an Annotation Assistant for ESC/Java", *FME 2001: Formal Methods for Increasing Software Productivity, International Symposium of Formal Methods Europe, Berlin, Germany, March 12-16, 2001, Proceedings*, 2001.

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6. After the filtering, go to step 2.

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$$TrInv(x, x') \subseteq TPA^{<n}(x, x')$$

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Evaluation

Comparison of Houdini and Non-Houdini performance (600 seconds timeout)

	Number of tests	split-TPA (Houdini)	split-TPA	TPA (Houdini)	TPA
LIA linear	585	342	332	302	295
LRA linear	498	202	196	136	130

Table: CHC-COMP'21 selection

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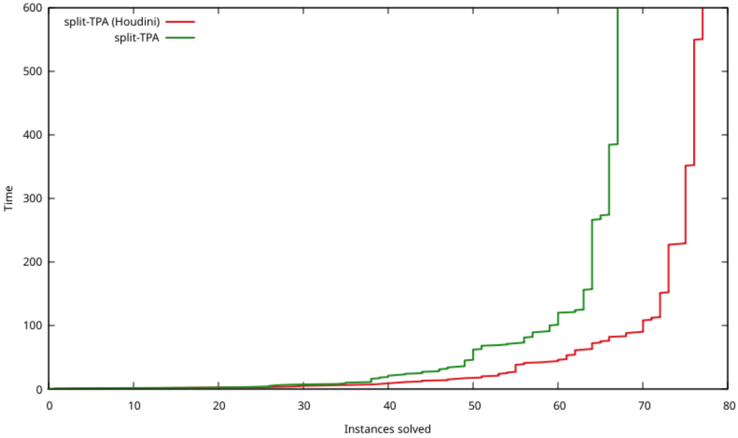

Future Work

- Introduce Houdini-based invariant search in exact transition abstractions
- Filter out low-potential candidates
- Improvements to the algorithm to pick the candidates for invariants

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Questions?

References I

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