Multi-Agent Path Finding with Continuous Time Using SMT($\mathcal{LRA}$)

15th Alpine Verification Meeting (AVM'23)

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Czech Academy of Sciences

Czech Technical University in Prague

12th September, 2023

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Multi-Agent Path Finding:

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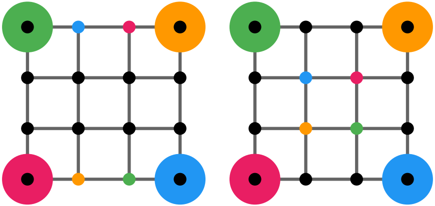

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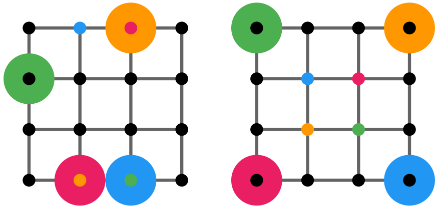

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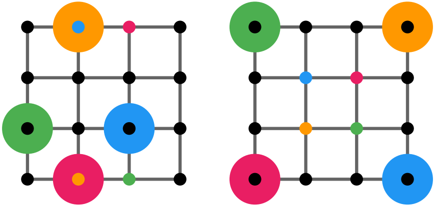

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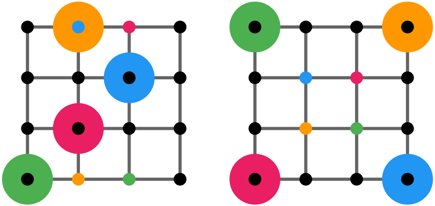

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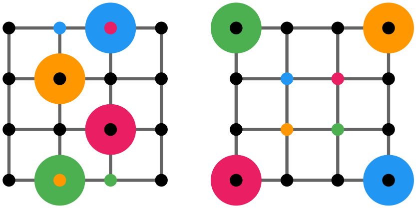

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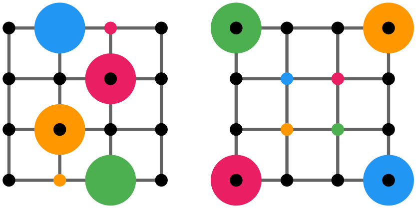

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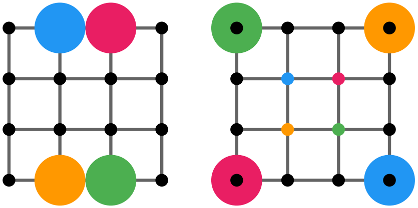

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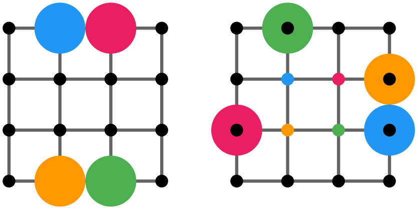

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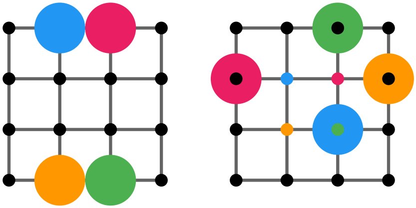

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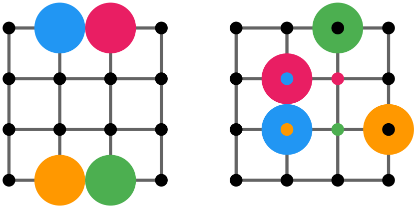

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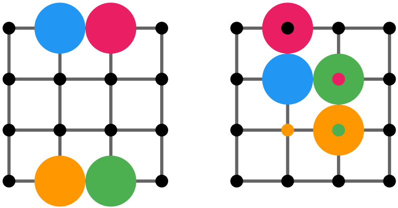

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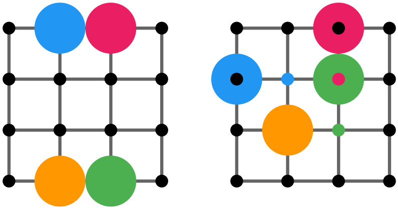

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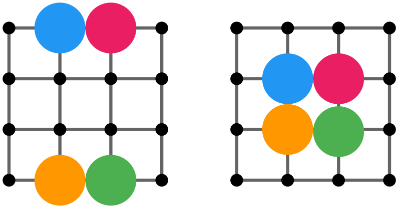

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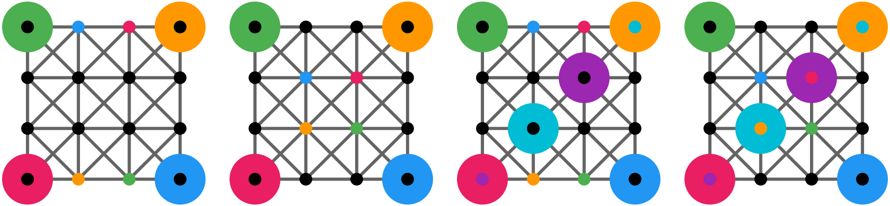

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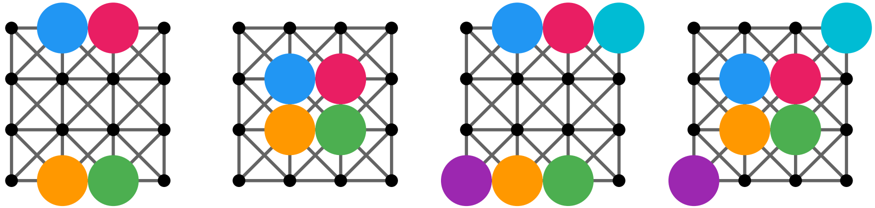

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Our approach

- We use **SAT** for efficient handling of **mutual exlusions** of agents.
- We use **SMT($\mathcal{LRA}$)** for efficient handling of **timing** constraints.
- We use **simulations** for evaluation of **conflict intervals** of agents.
- We relax the problem to finding **bounded suboptimal** plans.

Linear Real Arithmetic

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- Handles **linear** constraints on **rational** variables.
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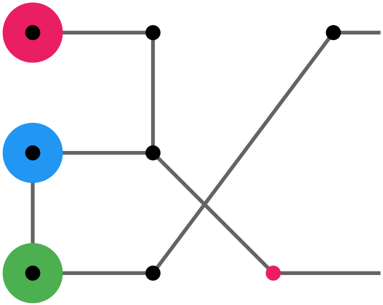
How we exploit $\mathcal{LRA}$

- We use $\mathcal{LRA}$ only for modeling time constraints of the agents.
- We **incrementally add conflict** interval constraints in a **lazy fashion**.

$\rightarrow$ Efficient **avoidance** of discovered conflicts.

Conflict Reasoning



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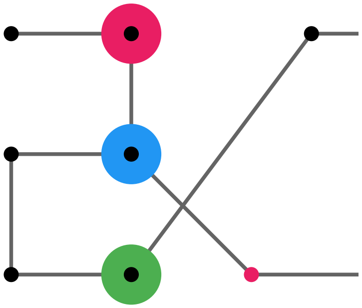

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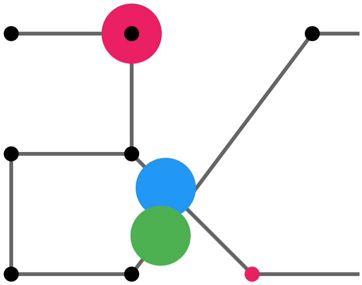

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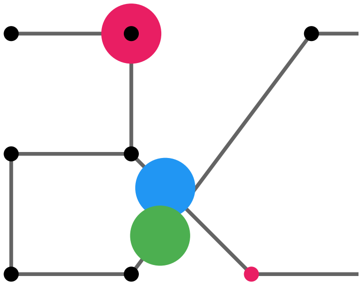

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$$\neg\Big(\mathit{blue}.V[1] = 5 \wedge \mathit{blue}.V[2] = 8 \wedge \mathit{green}.V[1] = 7 \wedge \mathit{green}.V[2] = 2$$
$$\wedge\ \mathit{blue}.T[1] - \mathit{green}.T[1] < \mathbf{3.743} - 2$$
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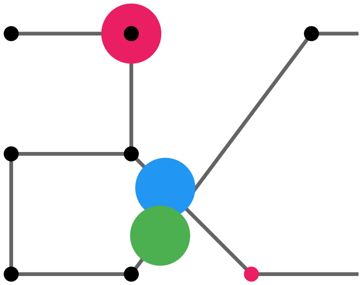

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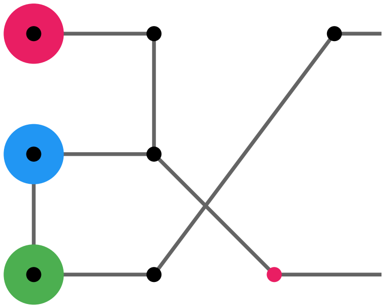

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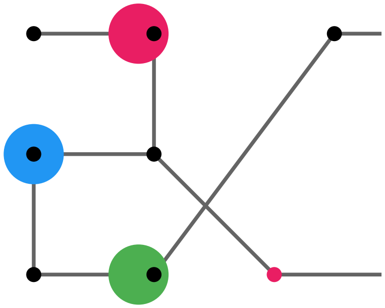

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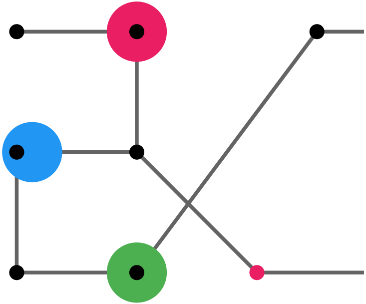

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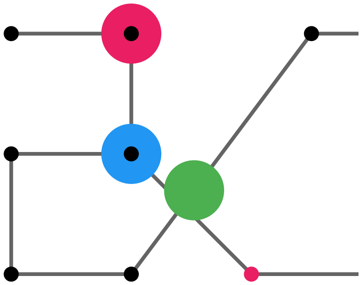

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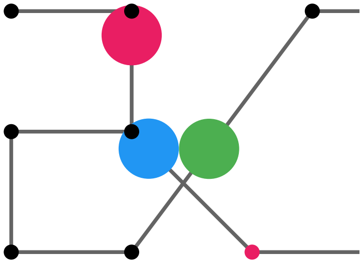

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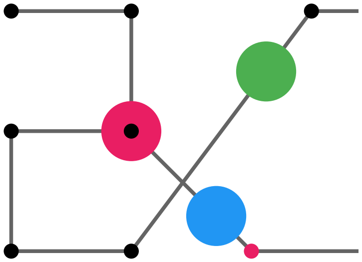

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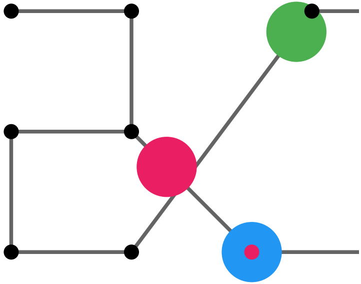

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→ Fast and **extensible** to more complicated and **realistic** motions.

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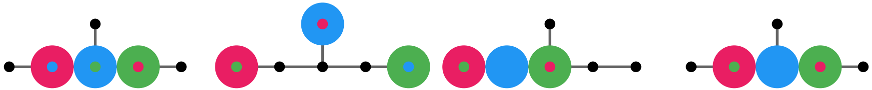
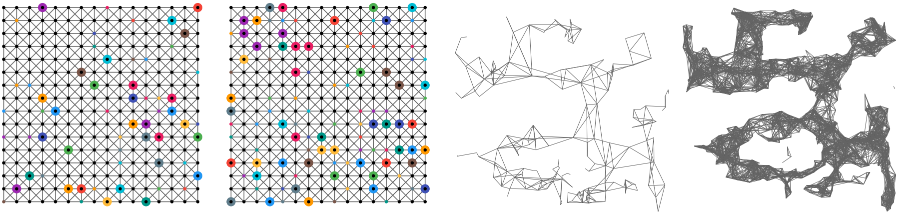
  - Offline planning vs. online execution.

More Demos . . .





Future Work

- Modeling dynamic phenomena of agents.
- Allowing trajectories of agents not to be just straight lines.
  - For example roadmaps with curves.
- ...

Questions?

$\mathrm{MAPF_R}$

https://gitlab.com/Tomaqa/mapf_r

https://github.com/Tomaqa/mapf_r-visualizer

UN/SOT: UN/SAT modulo ODEs Not SOT

https://gitlab.com/Tomaqa/unsot

tomaqa@gmail.com