NPRG075

Unexpected perspectives on types

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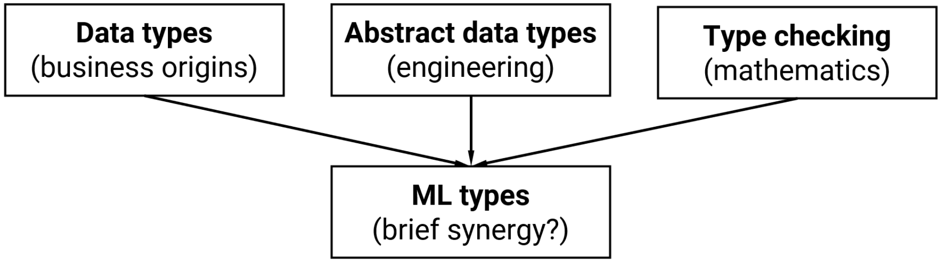
Lectures: Monday 12:20, S7
https://d3s.mff.cuni.cz/teaching/nprg075

Beyond types

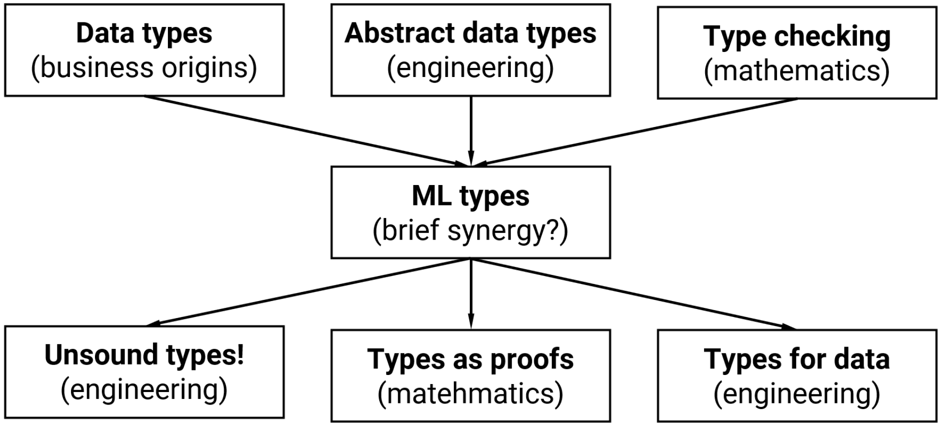
Recent developments



Convergences and divergences

ML brings together data types, abstract types and checking

End of the history?



Convergences and divergences

ML brings together data types, abstract types and checking

End of the history?

Developments in new directions in engineering and mathematics!

Types

Mathematical connections

Types

Mathematical connections

- ± Type constructors as algebraic operations
- → Proofs in propositional & predicate logic
- ◆ Linear logic and modal logics
- ⇄ Types and cartesian closed categories

Example

Are these two type definitions equivalent?

```
type Contact =
  | Email of string
  | Phone of digits
  | Both of string * digits

type Customer =
  { Name : string
    Contact : Contact }
```

Can one represent some values the other cannot?

```
type Option<'T> =
  | Some of 'T
  | None

type Customer =
  { Name : string
    Phone : Option<digits>
    Email : Option<string> }
```

Calculating with types

Type constructor algebra

- Record behaves as $A * B$ or $A \times B$
- Unions behave as $A + B$ or $A \cup B$
- Functions `A->B` behave as B^A
- Unit type is $\mathbf{1}$ and void (never) is $\mathbf{0}$

Usual algebraic laws work!

- $A * (B + C) = A * B + A * C$
- $A * \mathbf{1} = A$ and $A * \mathbf{0} = \mathbf{0}$

Calculating with types

$$Contact = (Phone * Email) + Email + Phone$$

$$\begin{aligned}
&Customer1 \\
&\quad = Name * Contact \\
&\quad = Name * ((Phone * Email) + Email + Phone)
\end{aligned}$$

$$\begin{aligned}
&Customer2 \\
&\quad = Name * (Phone + 1) * (Email + 1) \\
&\quad = Name * ((Phone + 1) * Email + (Phone + 1) * 1) \\
&\quad = Name * ((Phone * Email) + Email + Phone + 1)
\end{aligned}$$

What else works?

Binary trees

- Derivative of a binary tree?
- $btree = leaf + btree * btree$
- Treat $btree$ as the variable

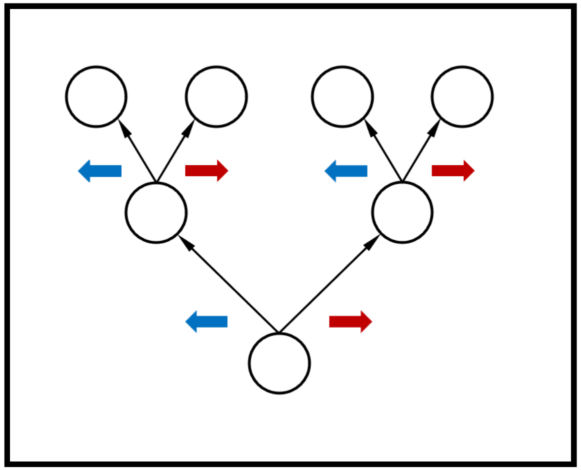
Derivatives

- Rules in case you forgot:
  tinyurl.com/nprg075-diff

Derivatives and inverses

Derivative of a binary tree

- $btree = leaf + (btree^2)$
- $btree' = 2 * btree$
- Steps for iterating over containers
- Recursively $\mathbf{2 * (2 * (2 * \ldots))}$

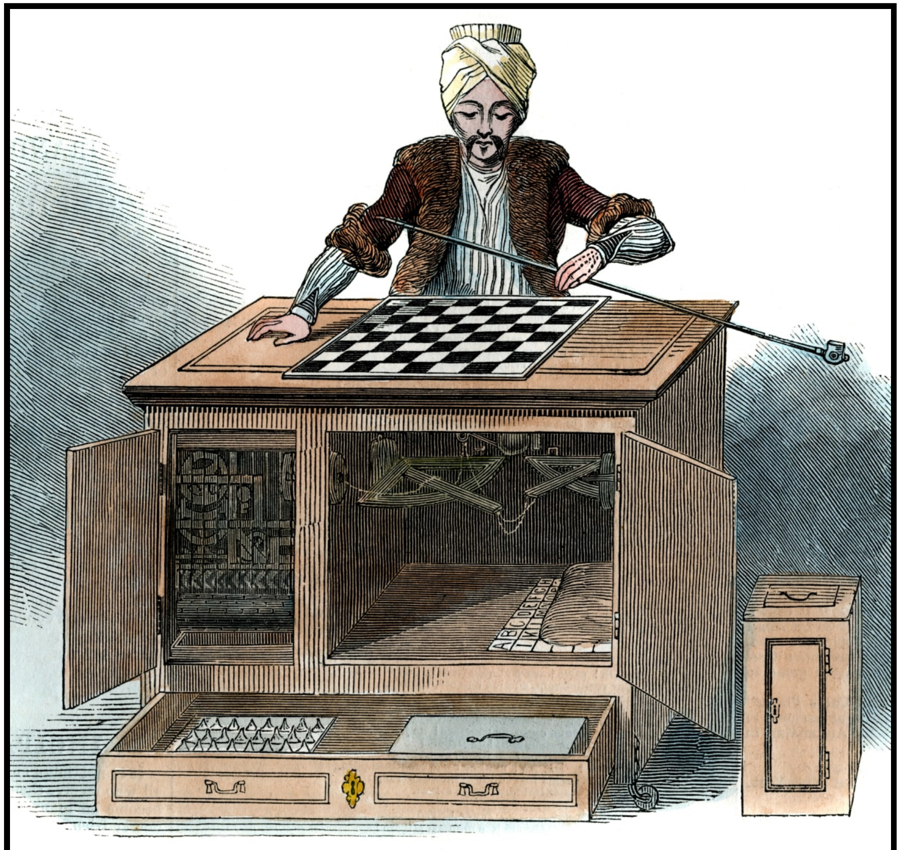


Can define the inverse!

- Works only in linear logic
- $A^{-1} = A \multimap \mathbf{1}$, i.e. a function that consumes a value
- $(A^{-1} \times A) \multimap \mathbf{1}$, i.e. one direction of equality

Types

Curry-Howard isomorphism


THE AUTOMATON CHESS PLAYER.

Miraculous link?

Types in programming are propositions in logic!

Programs are proofs!

Not that surprising..

Hard work to make it fit

Same origins in foundations of mathematics

Curry-Hoard isomorphism

Types as propositions

Function $A \to B$ corresponds to implication

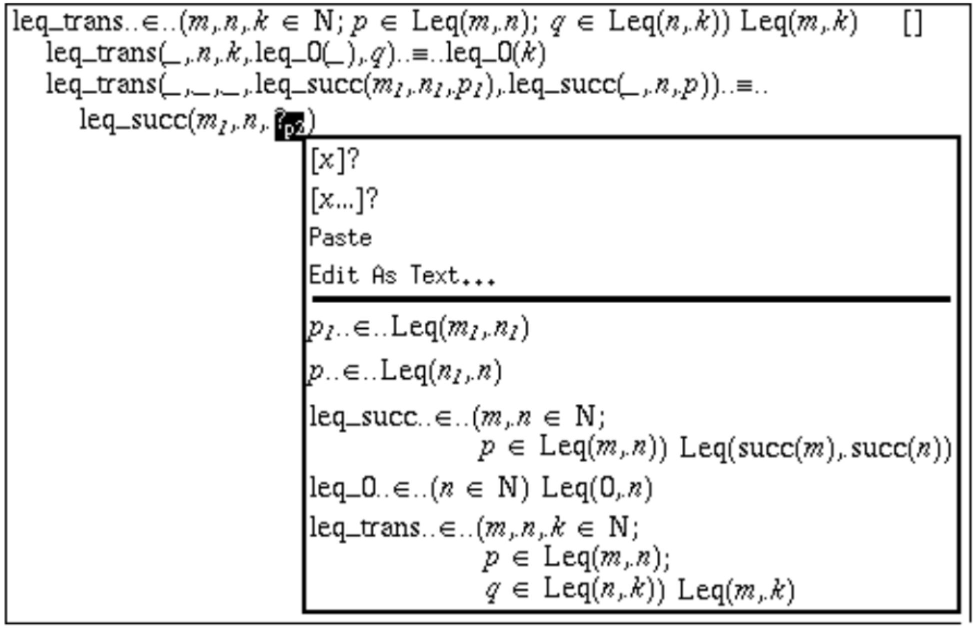
Product $A \times B$ corresponds to conjunction $A \land B$

Union $A + B$ corresponds to disjunction $A \lor B$

Proofs are programs

A well-typed program of type A is a proof of A

Write program to show that a property holds!



Theorem provers

Alf, Coq, Agda & more

Construct proofs by interactively creating programs

Show resulting program (Agda) or list of interactions (Coq)

Programs can run too

Programs as proofs

Function composition

Proposition: $((A \to B) \wedge (B \to C)) \to (A \to C)$

Program as proof: $\lambda(f, g).\lambda a.g(fa)$

Distributivity

Proposition: $A \wedge (B \vee C) \to (A \wedge B) \vee (A \wedge C)$

Program as proof: $\lambda(a, \mathbf{inl}\ b).\mathbf{inl}\ (a, b)$

$\lambda(a, \mathbf{inr}\ c).\mathbf{inr}\ (a, c)$

Inference rules for types and logic

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x . e : \tau_1 \to \tau_2} \qquad \frac{\Sigma, A \vdash B}{\Sigma \vdash A \to B}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash (e_1, e_2) : \tau_1 \times \tau_2} \qquad \frac{\Sigma \vdash A \qquad \Sigma \vdash B}{\Sigma \vdash A \wedge B}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \qquad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1\, e_2 : \tau_2} \qquad \frac{\Sigma \vdash A \to B \qquad \Sigma \vdash A}{\Sigma \vdash B}$$

Language design

Importing ideas via maths

- Simplifying types using algebraic laws
- Making sense of units and empty types
- Types inspired by linear and modal logic?
- Types for universal and existential quantifiers?

Linear types

$$\frac{}{x : \tau \vdash x : \tau}$$

$$\frac{\Gamma_1 \vdash e_1 : \tau_1 \to \tau_2 \qquad \Gamma_2 \vdash e_2 : \tau_1}{\Gamma_1, \Gamma_2 \vdash e_1\, e_2 : \tau_2}$$

$\lambda f.\, \lambda g.\, \lambda x.\ f(x) + g(x)$ no type

$f : \alpha \to num,\ g : \alpha \to num,\ x : \alpha$
$\vdash f(x) + g(x)$

Variable must be used exactly once!

Resource usage in programming!

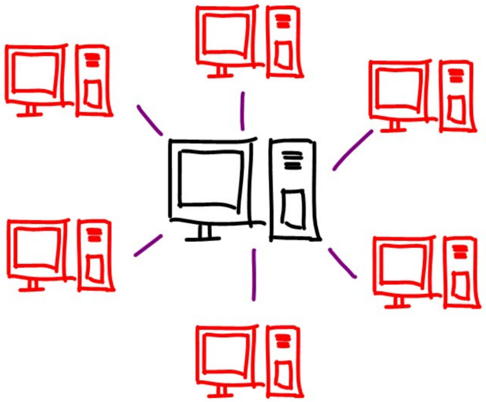
Avoid aliasing, efficient memory management

Generalizations to control sharing

Types for modal logics

Necessity and possibility

- $\diamond A$ - possibility - in a possible world
- $\square A$ - necessity - all possible worlds



Distributed systems

- Value A, address $\diamond A$, mobile code $\square A$
- Axiom $\square A \to A$ - run mobile code to get value
- Axiom $A \to \diamond A$ - take address of local value
- Axiom $\diamond A \to \square \diamond A$ - address is mobile

Dependent types

Quantifiers as type constructors

- Universal quantification $\Pi_{x:A} B(x)$
  Dependent function `(x:A) -> B(x)`
- Existential quantification $\Sigma_{x:A} B(x)$
  Dependent pair `(x:A) * B(x)`

Programming languages

- Origins in theorem provers
- Dependently-typed languages like Coq, Idris and Agda
- Some aspects expressible in Haskell, Scala

Using with dependent types

Capture precise information

Vector of a known length `Vec (n:int) A`

Other properties, like sortedness of a list

Programming with fancy types

Dependent pair and function

```
vectWithLength : (n:int) * Vec n string
initVector : (x:int) -> (v:A) -> Vec x A
```

Types

Engineering perspectives

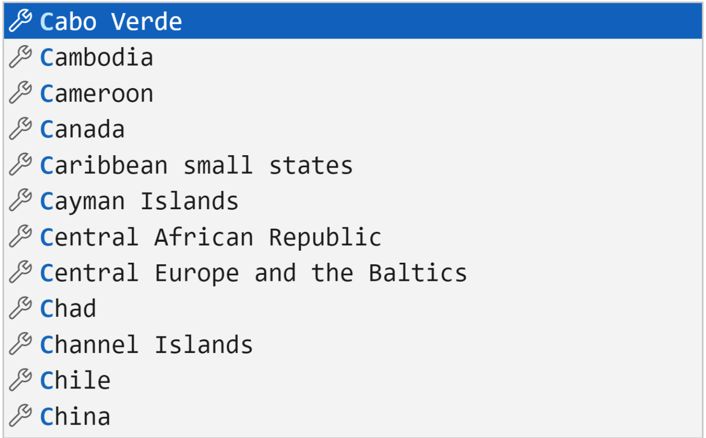
Demo

Checking weather in F#

Type providers

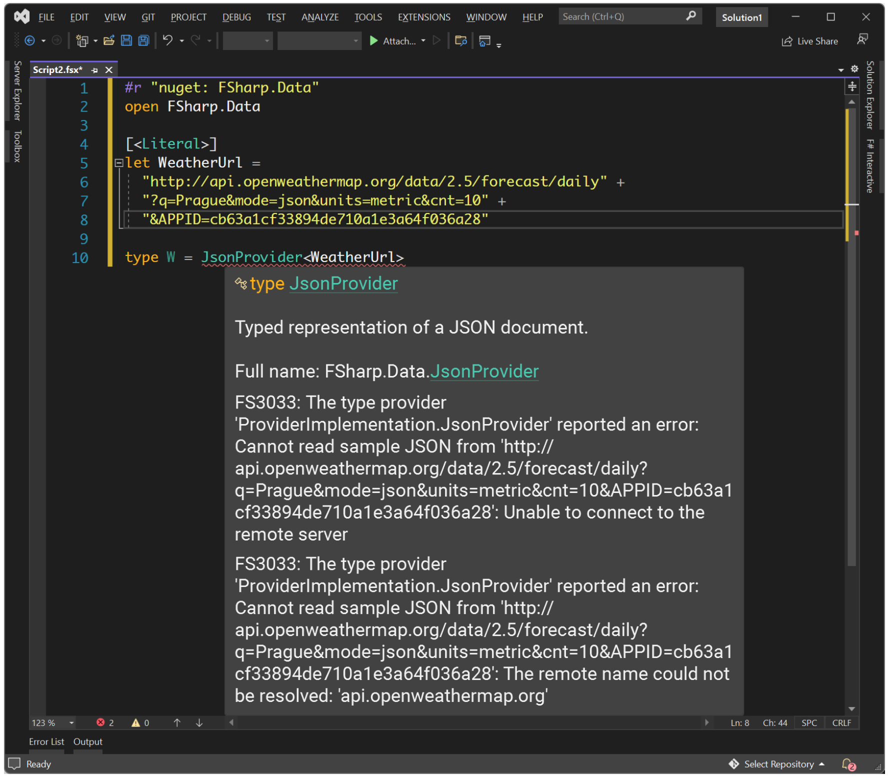
What is a type provider?

- Extension run at compile-time
- Can run arbitrary code
- Generates classes with members



What can they be used for?

- Infer structure of JSON, XML, CSV
- Import explicit database schema
- Interface with a foreign API



Static type checking?

Type error on a train!

More useful when external service changes format

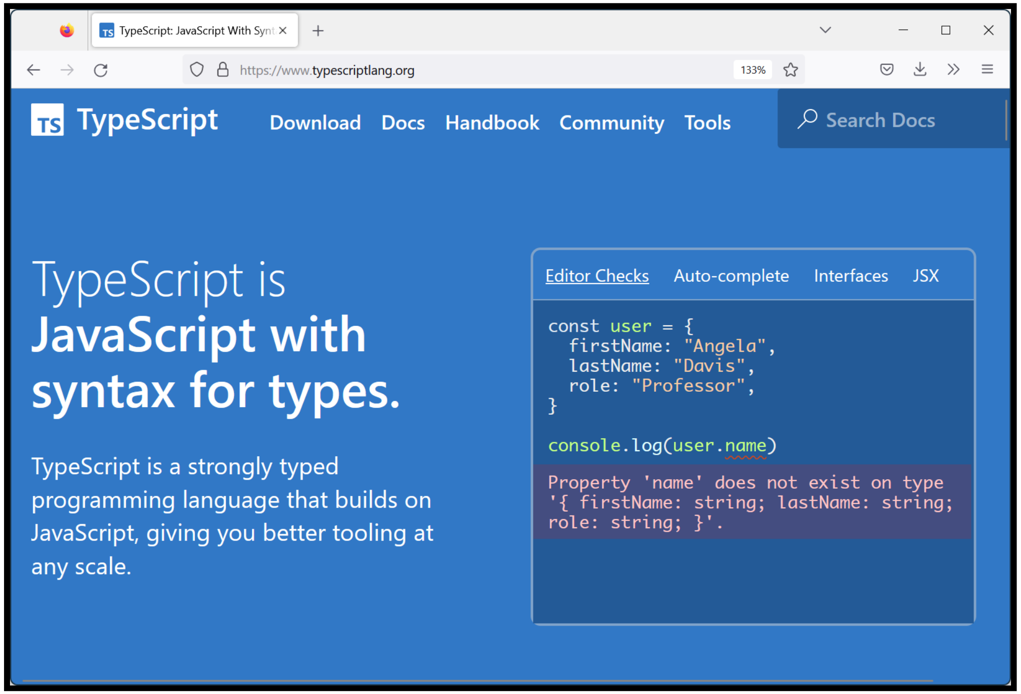
Well-typed programs do not go wrong?

Except when the world breaks assumptions about the schema

Types

Engineering perspective

- Types have to be useful, not always right
- Even unsound types help software engineers
- Invaluable for tooling (completion, checking)
- Documentation and structuring mechanism



TypeScript types

Unsound because of 'any', covariance, unchecked imports

Checking works well enough!

More reliable editor auto-completion

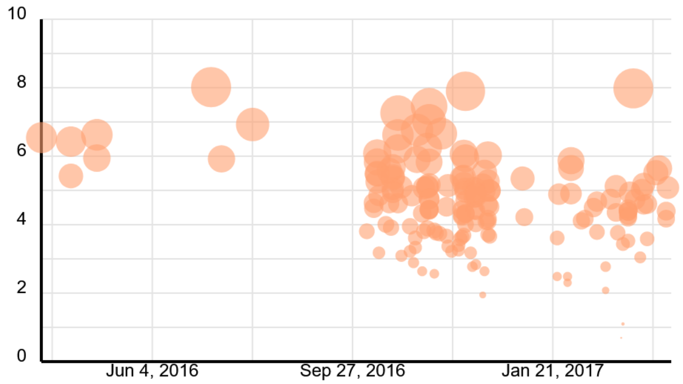
Demo

Type providers in The Gamma

The Gamma design

Iterative prompting

- Do everything via a type provider
- Construct SQL-like queries & more
- What are the limits of this?



Type provider tricks

- Lazy type generation for "big" types
- Parameterized (dependent) providers
- Fancy types for the masses

Fancy types for the masses

Row types

$$\frac{\Gamma \vdash e : [f_1 : \tau_1, \ldots, f_n : \tau_n]}{\Gamma \vdash e.\mathrm{drop}\ f_i : [f_1 : \tau_1, \ldots, f_{i-1} : \tau_{i-1}, f_{i+1} : \tau_{i+1}, \ldots, f_n : \tau_n]}$$

Embed as classes

$$\frac{\Gamma \vdash e : C_1}{\Gamma \vdash e.\mathrm{drop}\ f_i : C_2}$$

$$fields(C_1) = \{f_1 : \tau_1, \ldots, f_n : \tau_n\}$$
$$fields(C_2) = \{f_1 : \tau_1, \ldots, f_{i-1} : \tau_{i-1}, f_{i+1} : \tau_{i+1}, \ldots, f_n : \tau_n\}$$

Conclusions

Unexpected perspectives on types

Figure 1. A typical design process

Engineering and mathematical views

Complementary ways of designing & evaluating

Import ideas using maths, prove them correct

Adapt ideas for engineering purpose, show they work

Reading

When Technology Became Language:
The Origins of the Linguistic Conception of Computer Programming
From davidnofre.com or direct link

What to read and how

- The birth of programming languages
- Dramatic change in thinking!
- Longer, so read what you like...

Conclusions

Unexpected perspectives on types

- Many ideas imported through mathematics!
- Dependent, linear and modal types
- Making it work in practice is a challenge

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Type providers & related

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