Semantics - 2024

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$$e ::= x \mid \lambda x.e \mid e\,e \mid n \mid e + e$$
$$v ::= n \mid \lambda x.e$$

$\boxed{\text{evaluation}}$

$$
\begin{aligned}
& (\lambda x.\ x + 10)\ (1+2) \\
\to\ & (\lambda x.\ x + 10)\ \ 3 \\
\to\ & 3 + 10 \\
\to\ & 13
\end{aligned}
$$

$\boxed{\text{reduction rules}}$

$$\frac{n_3 = n_1 + n_2}{n_1 + n_2 \to n_3}\ (\text{add})$$

$$\frac{e_1 \to e_1'}{e_1 + e_2 \to e_1' + e_2}\ (\text{add1})$$

$$\frac{e_2 \to e_2'}{n_1 + e_2 \to n_1 + e_2'}\ (\text{add2})$$

$$\frac{}{(\lambda x.e_1)\ v_2 \to e_1[x \leftarrow v]}\ (\text{app})$$

$$\frac{e_1 \to e_1'}{e_1\ e_2 \to e_1'\ e_2}\ (\text{app1})$$

$$\frac{e_2 \to e_2'}{(\lambda x.e_1)\ e_2 \to (\lambda x.e_1)\ e_2'}\ (\text{app2})$$

$\boxed{\text{example revisited}}$

$$\frac{\dfrac{3 = 1+2}{(1+2) \to 3}\ (\text{add})}{(\lambda x.\ x + 10)\ \ (1+2) \to (\lambda x. x + 10)\ 3}\ (\text{app2})$$

$$\frac{}{(\lambda x. x + 10)\ 3 \to 3 + 10}\ (\text{app})$$

$$\frac{13 = 3 + 10}{3 + 10 \to 13}\ (\text{add})$$

$$\frac{13 = 3+10}{3+10 \to 13}\ (\text{add})$$

conditionals

$e ::= \ldots \mid \text{if } e_1 \text{ then } e_2 \text{ else } e_3$

$$\frac{e_1 \to e_1'}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3 \to \text{if } e_1' \text{ then } e_2 \text{ else } e_3}\ (\text{cond1})$$

$$\frac{n = 0}{\text{if } n \text{ then } e_2 \text{ else } e_3 \to e_3}\ (\text{cond2})$$

$$\frac{n \neq 0}{\text{if } n \text{ then } e_2 \text{ else } e_3 \to e_2}\ (\text{cond})$$

state

$e ::= \ldots \mid \ell \mid \ell := e$

$\langle e, s \rangle \qquad s : \mathbb{L} \to \mathbb{V}$

$\langle \ell, s \rangle \to \langle s(\ell), s \rangle$

$\langle \ell := v, s \rangle \to \langle v, s \cup \{\ell \mapsto v\} \rangle$

reactive

$e ::= \ldots \mid \varepsilon!e \mid \varepsilon.\text{sub}(\lambda x.e)$

$\varepsilon.\text{sub}(\lambda x.\ \varepsilon!(x*10+1);\ \varepsilon!(x*10+2))$

$$\langle e_1 \mid e_2 \mid e_3 \mid \dots \rangle, h$$

$$\frac{}{\langle v \mid e_1 \mid e_2 \mid \dots \rangle, h \to \langle e_1 \mid e_2 \mid \dots \rangle, h}$$

$$\frac{e_1 \to e_1'}{\langle e_1 \mid e_2 \mid \dots \rangle, h \to \langle e_1' \mid e_2 \mid \dots \rangle, h}$$

$$\frac{e = h(\varepsilon)[x \leftarrow v]}{\langle \varepsilon ! v \mid e_1 \mid e_2 \mid \dots \rangle, h \to \langle e \mid e_1 \mid e_2 \mid \dots \rangle, h}$$

$$\to \langle e_1 \mid e_2 \mid \dots \mid e \rangle, h \quad ?$$