NPRG077

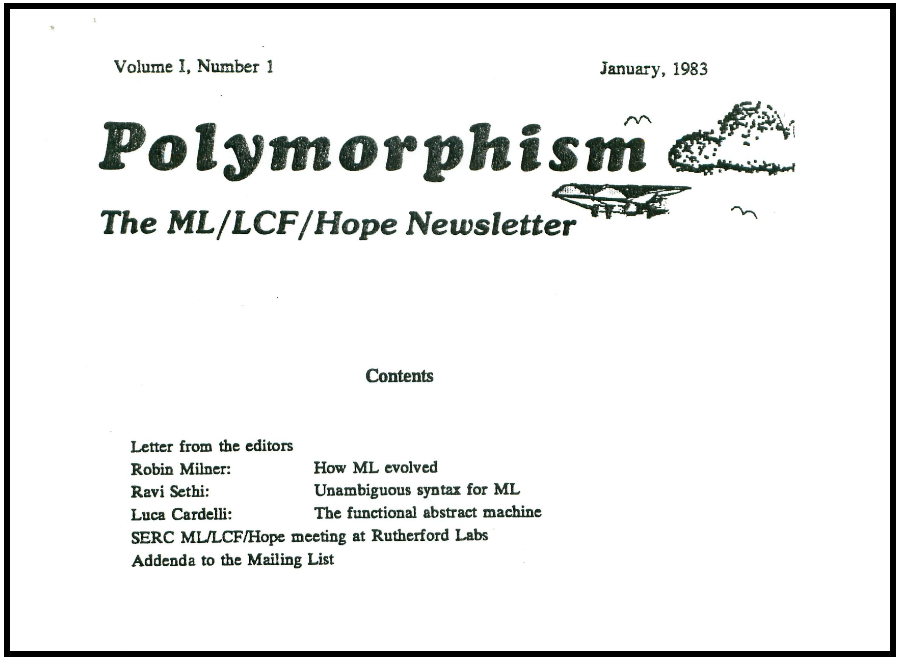
TinyHM: Tiny Hindley-Milner type inference

Tomáš Petříček, 309 (3rd floor)
petricek@d3s.mff.cuni.cz
https://tomasp.net | @tomaspetricek
https://d3s.mff.cuni.cz/teaching/nprg077

Not a programming system!?

- An important part of the ML experience
  Makes ML practical and OCaml efficient
- Learn some subtle aspects of F# type inference
  Some discovered late through proofs and errors
- Good example of constraint solving...
  Important technique, used in Prolog & elsewhere

Origins of ML



Volume I, Number 1 January, 1983

Polymorphism

The ML/LCF/Hope Newsletter

Contents

LCF theorem prover

ML used for writing meta-programs to generate proofs

Types used to ensure the validity of proofs

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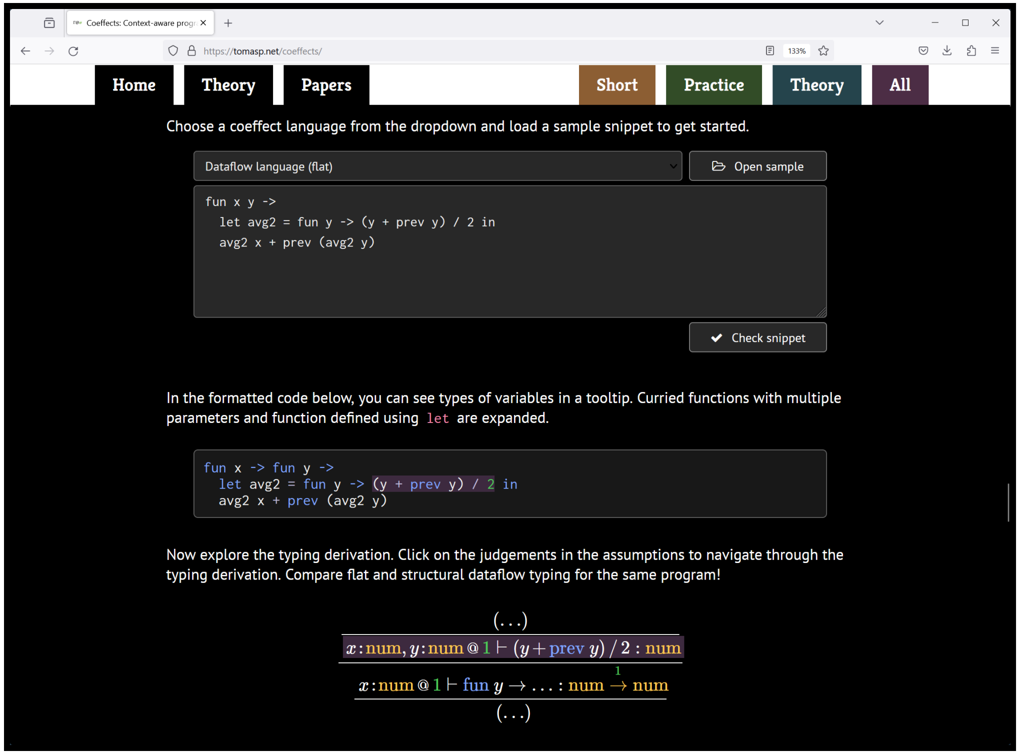
Hindley-Milner

A brief history of type inference

- **Hindley (1969)** for Combinatory Logic
- **Milner (1978)** for ML with polymorphism
- **Damas (1985)** with formal analysis and proofs
- **Since then** - type classes, other extensions



Demo Coeffects playground

Constraint solver code on GitHub

ML type inference

How does F# figure out the types?

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Demo

Basic type inference in F#

How F# type inference works

Constraint-based

- Collect & solve constraints
- No annotations needed for ML!

```
let twice f x = f (f x)
    val twice: f: ('a -> 'a) -> x: 'a -> 'a

    Full name: Inference.twice
```

Let polymorphism

- Infer generic type of let-bound functions

Limitations in ML and F#

- Value restriction for generic values
- Harder to deal with .NET objects

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Demo

Type inference limitations in F#

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A bit of theory

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Type systems

Typing rules

Given a typing context Γ, the expression e has a type τ

$$\Gamma \vdash e : \tau$$

The problem in general

We know some of these, want to figure out the rest

Type systems

Type checking

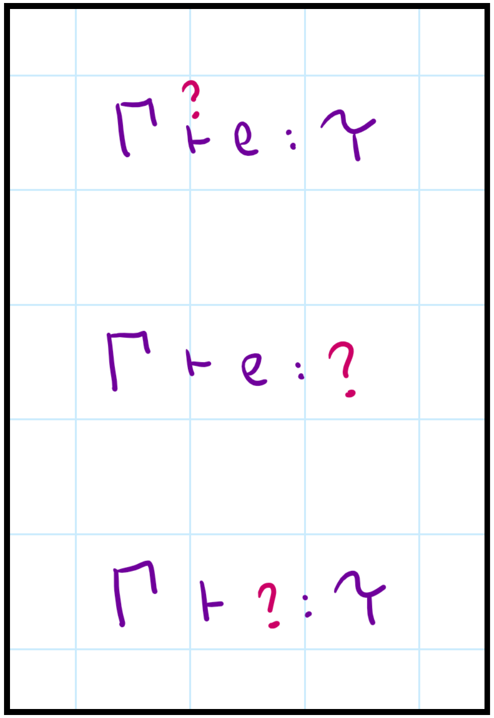
- Know it all. Check derivation exists!
- Easy for syntax-driven rules

Type inference

- Know expression. Figure out the type!
- Ideally most general (best) type

Program synthesis

- Not typical setting, but for completeness...



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Principal type (most general)

Type τ is the most general type for an expression e in context Γ if

- $\Gamma \vdash e : \tau$ and
- $\forall \sigma .\ \Gamma \vdash e : \sigma \rightarrow \sigma <: \tau$

Best type of an expression

Any other type of the expression is a special case (subtype) of it

Type inference

- ✓ How Hindely-Milner type inference works?
  Produces most general type (for ML)
- ✗ How Hindely-Milner type inference breaks?
  Nominal types with members, interfaces, etc.
- ? Alternative methods for type inference
  Bidirectional - combines checking and inference

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Constraint generation & solving

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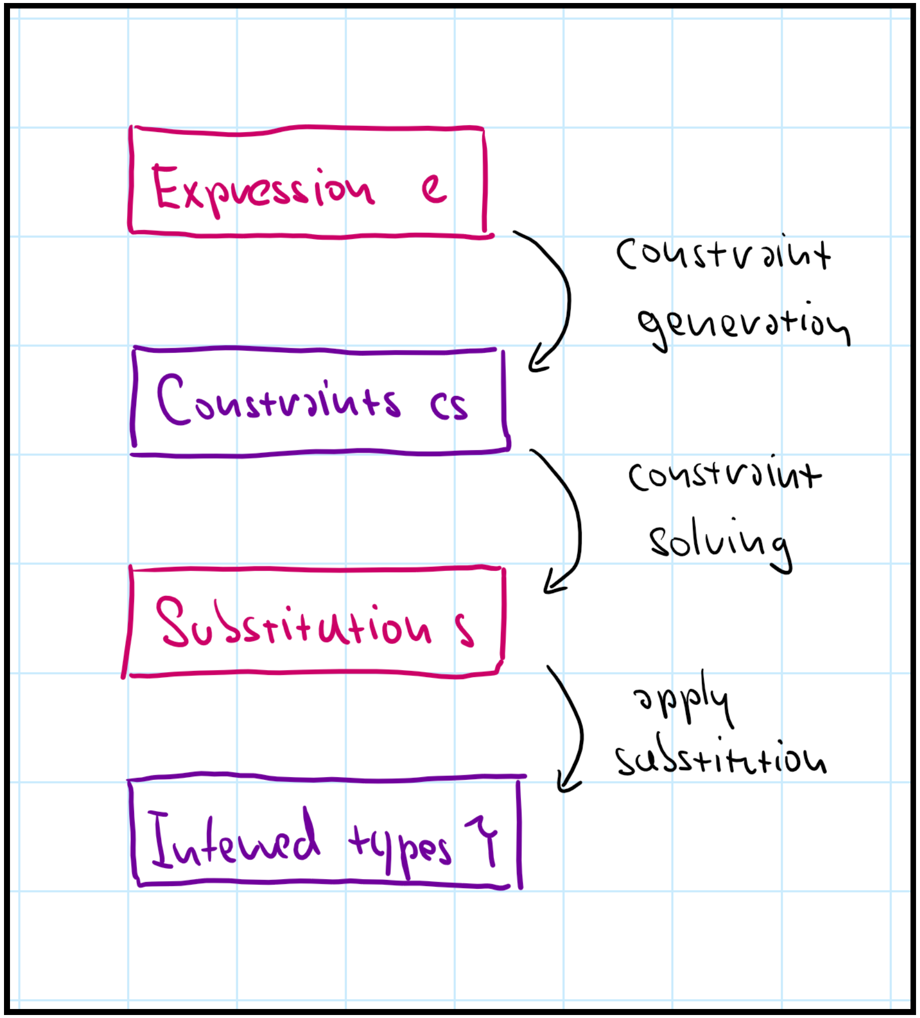

Two phase process

Generate constraints

Recursively over expression

Solve constraints

Recursively over constraint set

In the "Algorithm W", the two are combined. We separate them!

What is a constraint?

```
(* Basic types with
   type variables *)
type Type =
  | TyNumber
  | TyVariable of string
  | TyFunction of Type * Type
  | TyList of Type

(* Constraint specifies
   that one type should be
   unified with another *)
type Constraint =
  Type * Type
```

A pair of types that should be unified

Easy or impossible

```
int = int -> int
int list = int list
```

Tricky with variables

```
'a = int -> 'b
'a = 'c -> int
```

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Constraint generation

- Generate type and constraints recursively
- Generate new fresh type variables as needed
- Variables with new type variables in context
- Most checking done via constraints

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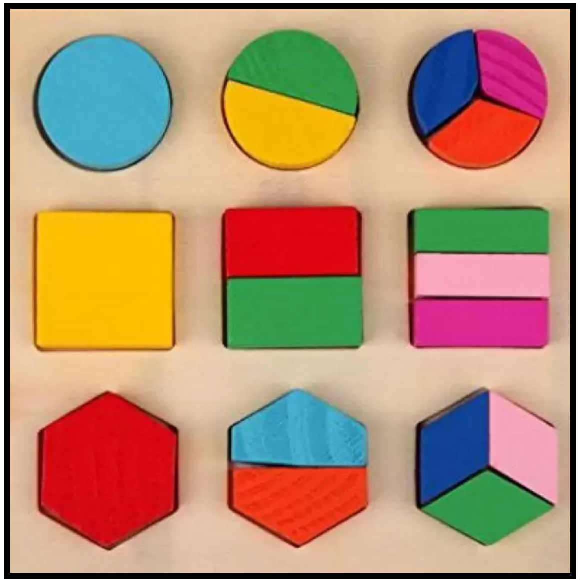
Sketch

Generating constraints

Constraint solver structure

Simplest possible example

- Peano numbers: `Zero`, `Succ(x)`
- Equality constraints with variables
- e.g. `Succ(x) = Succ(Succ(Zero))`



Creating a solver

- Discharge matching constraints
- Fail on mismatching constraints
- Generate more for matching nested
- Needs to handle substitutions...

Demo

Solving numerical constraints

```
let rec solve constraints =
  match constraints with
  | [] -> []
  | (Zero, Zero)::cs -> solve cs
  | (Succ n1, Succ n2)::cs -> solve ((n1, n2)::cs)
  | (Zero, Succ _)::_ | (Succ _, Zero)::_ ->
      failwith "cannot be unified"
  | (n, Variable v)::cs | (Variable v, n)::cs ->
      let substs = solve cs
      (v, n)::substs
```

- #1 apply all 'substs' to 'n'
- #2 substitute 'n' for 'v' in remaining constraints
- #3 check that 'v' does not appear in 'n'

Remaining work

Substitution (#1)

Replace variable in remaining constraints

Substitution (#2)

Apply substitutions to assigned type

Occurs check (#3)

Check for unsolvable constraints

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Demo

Substitutions and occurs check

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Inference code structure

Types supported

```
(* All possible types you may
   support: type variables,
   primitives and composed *)
type Type =
  | TyVariable of string
  | TyBool
  | TyUnit
  | TyNumber
  | TyFunction of Type * Type
  | TyTuple of Type * Type
  | TyUnion of Type * Type
  | TyList of Type
  | TyForall of string * Type

(* Types of known variables *)
type TypingContext =
  Map<string, Type>
```

Type variables

For constraint solving!

Primitive types

Match/mismatch

Composed types

Generate one or two new constraints

Polymorphic type

Forall (bonus)

Type inference operations

```
(* Given a list of
   constraints, produce a
   list of substitutions *)
val solve :
  list<Type * Type>
  -> list<string * Type>

(* Given a typing context
   (known variables) and
   expression, return the type
   of the expression and
   list of constraints *)
val generate :
  TypingContext
  -> Expression
  -> Type * list<Type * Type>
```

Constraint solving

Takes constraints

Produces substitution

Constraint generating

Takes an expression

Produces constraints

Also check variables

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Lab overview

Tiny Hindley-Milner step-by-step

TinyHM - Basic tasks

1. Complete the simple numerical constraint solver
   Add the two missing substitutions to make it work!
2. Solving type constraints with numbers and Booleans
   Follow the same structure, but now for type constraints...
3. Type inference for binary operators and conditionals
   Add constraint generation for a subset of TinyML
4. Supporting more TinyML expressions
   Add let, functions, application and occurs check
5. Adding simple data types
   Constraint generation for tuples

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TinyHM - Bonus & super tasks

1. Supporting more TinyML data types
   Add type checking for discriminated unions
2. Type inference for lists - poor method
   Add recursion & units and try this on list code!
3. Adding proper support for generic lists
   New type, but without explicit type declarations
4. Inferring polymorphic code for let bindings
   Implementing proper Hindley-Milner let-polymorphism
5. Exploring pathological cases
   Did you know HM has DEXPTIME complexity?

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Closing

Tiny Hindley-Milner type inference

Conclusions

Tiny Hindley-Milner type inference

- A remarkable quality of ML language(s)
- Cannot expect users to write types by hand!
- Nice introduction to constraint solving
- Much more can be done with this idea...

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