

# Decision Procedures and Verification

## Seminar 2

- (1 point) Consider the following CNF formula:  $(x_5 \vee \neg x_1 \vee x_3) \wedge (\neg x_3 \vee x_4) \wedge (\neg x_5 \vee x_1) \wedge (x_6 \vee x_1) \wedge (\neg x_1 \vee x_2) \wedge (\neg x_3 \vee \neg x_4) \wedge (\neg x_5 \vee \neg x_6) \wedge (x_3 \vee \neg x_4) \wedge (x_3 \vee x_5)$   
Simulate CDCL on the formula. Draw implication graphs. Derive conflict clauses.
- (0.5 points) Prove that in the conflict graph, the notion of a first UIP is well-defined, i.e. there is always a single UIP closest to the conflict node.
- (1 point) What is the worst-case space complexity of CDCL algorithm?
  - without clause learning
  - with clause learning
  - with clause learning where learned clauses have at most  $k$  literals
- (1 point) Consider following restriction in CNF formulae:
  - Every clause contains at most one positive literal
  - Every clause contains at most two literals

Suggest efficient algorithms for satisfiability testing of formulae in above cases.
- (1 point) (**incremental satisfiability**) Given two CNF formulas  $C_1$  and  $C_2$ , under what conditions can a conflict clause learned while solving  $C_1$  be reused when solving  $C_2$ ? In other words, if  $c$  is a conflict clause learned while solving  $C_1$ , under what conditions is  $C_2$  satisfiable if and only if  $C_2 \wedge c$  is satisfiable? *Hint*: think of CNF formulas as sets of clauses.