Abstraction

http://d3s.mff.cuni.cz

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faculty of mathematics and physics

Motivating example

```
int sum(int from, int to) {
  int total = 0;
  for (int i = from; i <= to; i++) {
    total += i;
  }
  return total;
}

main() {
  int x = sum(1, 1000);
  assert(x > 0);
}
```

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Abstraction

- Goal: smaller reachable program state space
- Approaches
  - Reducing the size of variables' data domains
  - Ignoring concrete values of certain variables
- Benefits
  - Mitigating the state space explosion
  - Improved scalability (performance)

Data abstraction

- Using abstract domains for program variables
- Tracking only abstract states of the program

- Abstract state = set of concrete states

- Process: mapping concrete to abstract
  - data types, values, operations, program states

Example: Signs abstraction

- Abstract data type
  - `int` → { NEG, ZERO, POS }

Q: What about values and operations ?

Let's consider only addition here.

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Example: Signs abstraction

- Abstract data type
  - `int` → { NEG, ZERO, POS }
- Abstract values
  - $\alpha(x) \subseteq$ { NEG, ZERO, POS}
- Abstract operation +

	NEG	ZERO	POS
NEG	{ NEG }	{ NEG }	{ NEG, ZERO, POS }
ZERO	{ NEG }	{ ZERO }	{ POS }
POS	{ NEG, ZERO, POS }	{ POS }	{ POS }

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Construction of abstract programs

- Transformation of program source code

```
int x = 0;

...

int y;
y = x + 2;
```

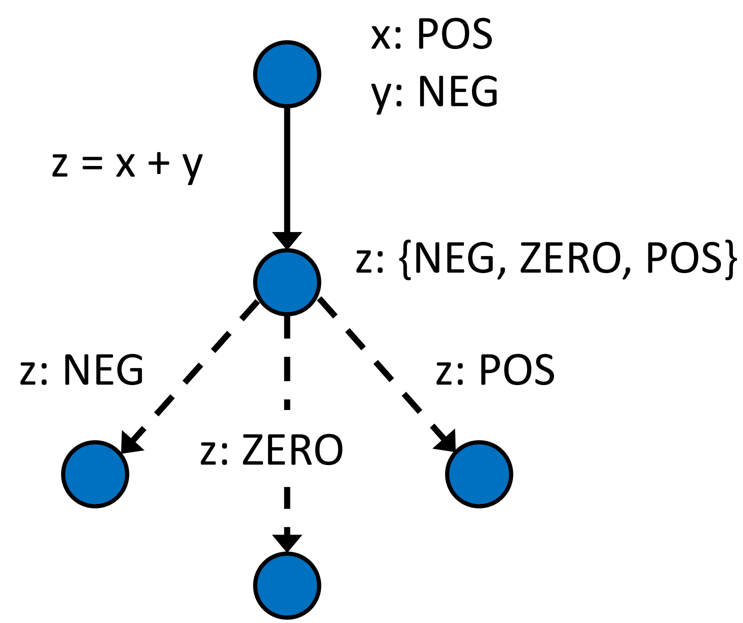
↔

```
Signs x = Signs.ZERO;

...

Signs y;
y = Signs.add(x, Signs.POS);
```

Abstract state space

- Non-deterministic choice
  - assignment, branching condition (if-else, loops)

```
int x = 5;
int y = -2;
z = x + y;
```



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Other simple data abstractions

- Interval abstraction
  - Example: $x < 0$, $0 \leq x \leq 10$, $x > 10$
- Combining intervals with concrete values
  - Example: $x < 0$, $x = 0$, $x = 1$, $x = 2$, $x = 3$, $x = 4$, $x > 4$

Predicate abstraction

Predicate abstraction

- Data type
  - Predicates about program variables
    - Theories: linear integer arithmetic, equality, arrays
    - Example: $x = 0$, $x > 0$, $y + z \geq 2$, $u = v$, $select(a,1) = 5$

- Abstract state
  - Some valuation of all the predicates

Example

```
int sum(int from, int to) {
   int total = 0;
   for (int i = from; i <= to; i++) {
      total += i;
   }
   return total;
}

int x = sum(1, 1000);
assert(x > 0);
```

Q: what predicates should we use here ?

Boolean program

```
bool P1 = false;
bool P2 = false;

// int total = 0;
P2 = true;

// int i = from;
P1 = *;

// total += i;
if (P1 && P2) P2 = true;
else P2 = *;
```

Predicates

P1: $i > 0$

P2: $total \geq 0$

Deriving predicate valuations

- Weakest preconditions
  - Predicate *p*: total ≥ 0
  - Statement *s*: total += i;
  - WP(*s*,*p*) ≡ total + i ≥ 0

- Querying the SMT solver
  - Example: p1 && !p2 → WP(*s*,*p*) is valid ?

- Processing results

  1) p1 && !p2 → WP(*s*,***p***) is valid ➔ `if (p1 && !p2) p = true;`
  2) p1 && !p2 → WP(*s*,**!*p***) is valid ➔ `if (p1 && !p2) p = false;`
  3) both valid or none valid ➔ `if (p1 && !p2) p = *;`

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Optimizations

- Goal: reduce the number of queries for SMT

- Possible approaches
  - Compute new valuation only for predicates that refer to variables modified by the given concrete assignment statement
    - We must be very careful though: aliasing
  - For generating branches of the big `if-else` statements in the abstract boolean program, consider only predicates that refer to variables read by the assignment statement

Verification using predicate abstraction

- Using model checker for boolean programs
  - Much easier task than for general programs (C, Java)
  - Well-known optimizations: symbolic model checking

- Practical challenges
  - Translating counterexamples back to source code
  - Encoding properties into reachability of assertions

Abstraction: characteristics

Abstraction: characteristics

Assume that we want to verify a given program.

Q: What important characteristic should the abstract program have ?

Over-approximation

- Abstract program captures all possible behaviors of the original concrete program
  - Behavior: possible control flow path, thread interleaving
- Purpose: complete verification (all reachable states)
- Examples
  - Simple data abstraction
  - Predicate abstraction
- Problem: imprecise abstraction
  - Captures some infeasible execution paths → **spurious errors**
  - Branch conditions replaced with a non-deterministic choice

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Abstraction: characteristics

Q1: Is there some other way to creating abstract programs than over-approximation ?

Q2: If yes, when does it make sense to use it ?

Under-approximation

- Abstract program captures only a certain subset of all possible behaviors of the concrete program
  - selected thread interleavings, reduced data domains
- Purpose: fast error detection (subset of reachable states)
- Examples
  - State space traversal with heuristics
  - Context-bounded search (traversal)
  - Bounded model checking in general
- Problem: imprecise abstraction
  - Omits some feasible execution paths ➔ missed errors

Abstractions: characteristics

Over-approximation	*Under-approximation*
Error in abstraction ↑ Error in concrete program	Error in abstraction ↓ Error in concrete program
Error-free abstraction ↓ Error-free concrete program	Error-free abstraction ↑ Error-free concrete program
Spurious errors	**Missed errors**

Abstraction: issues

- Very hard to get right
  - Too precise → state explosion
  - Too coarse → spurious errors

- Possible remedy
  - Start with coarse abstraction
  - Employ iterative refinement

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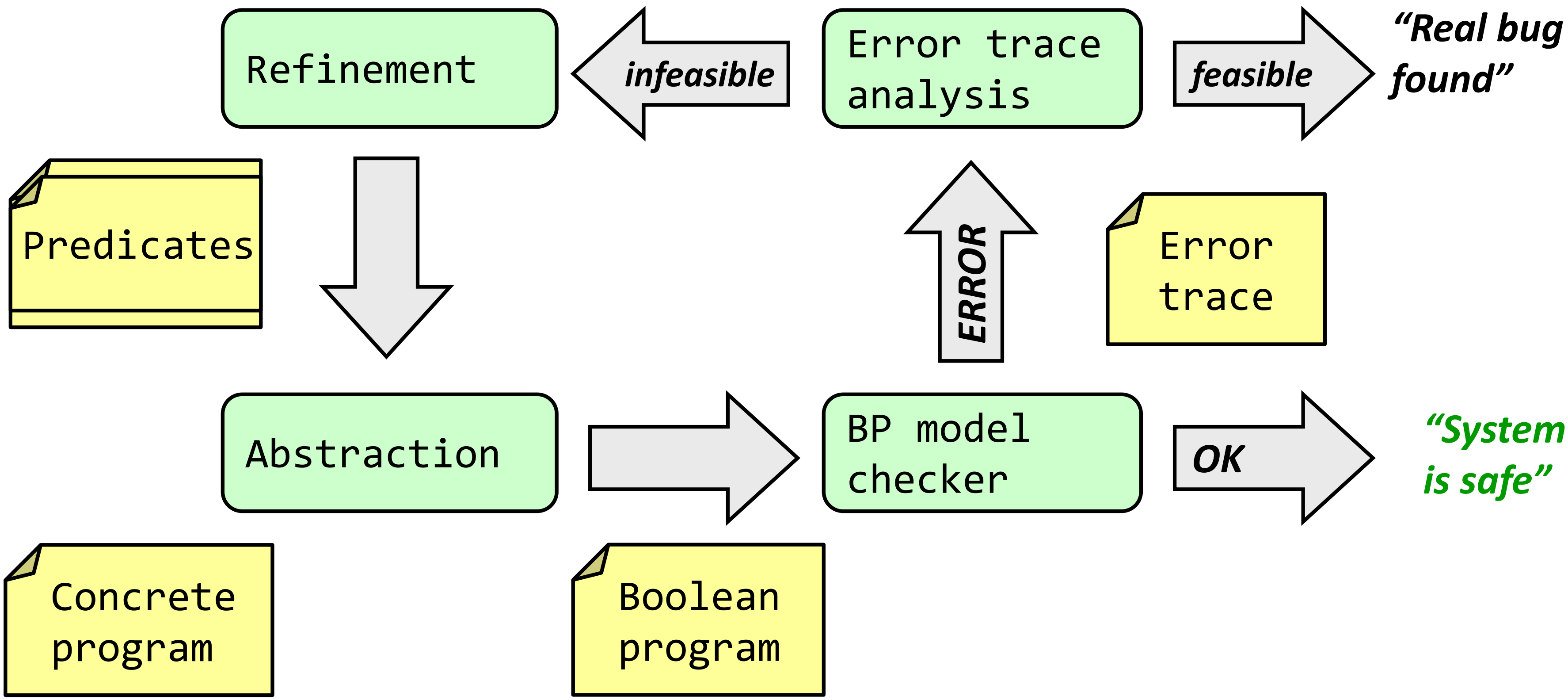
CEGAR

CEGAR

Counter-Example Guided Abstraction Refinement

- Automated iterative refinement based on spurious errors

CEGAR



Picture created by Ondřej Šerý

Challenges

- Checking error trace feasibility
- Inferring additional predicates

Checking error trace feasibility

- Record the path condition *PaC* using symbolic execution
  - Options selected at choice points (if-else, loops, non-determinism)
- Create path formula that encodes the whole error trace
  - The `assume` statement: clauses from the *PaC* (selected branches)
- Check satisfiability of the path formula (query the SMT solver)
- Example
  - Error trace

    ```
    index = 1; total = total + index; assume index > 1000
    ```
  - Path formula

    ```
    (index0 = 1) && (total1 = total0 + index0) && (index0 > 1000)
    ```

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Inferring additional predicates

- Divide path formula ϕ into two parts ϕ^- and ϕ^+
  - such that ϕ^- && ϕ^+ is unsatisfiable
- Then derive a Craig interpolant ψ for ϕ^- and ϕ^+
  - Logic formula ψ such that
    - $\phi^- \rightarrow \psi$, ϕ^+ && ψ is unsatisfiable, and
    - ψ uses symbols common to ϕ^- and ϕ^+
- Finally generate additional predicates from ψ

- Example
  - Path formula
    - `(index0 = 1) && (total1 = total0 + index0) && (index0 > 1000)`
  - ϕ^-: `index0 = 1 && total1 = total0 + index0`
  - ϕ^+: `index0 > 1000`
  - ψ: `index0 = 1` // newly inferred predicate in this case

- Disclaimer
  - Bad choices of inferred predicates may lead to non-termination
  - Tools generate predicates that may look strange (not intuitive)

SLAM/SDV

- Static Driver Verifier (SDV)
  - SLAM: verification engine that uses CEGAR
- Purpose
  - Analyzing third party Windows device drivers
    - Specific rules about proper usage of Windows kernel API
    - Major source of kernel crashes (infamous "blue screens")
    - Drivers have feasible code size and a strict environment
- Many extensions developed in the last decade
- Additional information
  - https://www.microsoft.com/en-us/research/project/slam/
  - Many research papers, slides, download, user guides

Optimizations

- Lazy abstraction
  - Set of predicates specific to each code location
  - Tools: BLAST

- Method summaries
  - Logic formula relating inputs and outputs
  - Summaries computed using interpolants
  - Tools: Whale, FunFrog, ...

Tools

- BLAST
  - http://mtc.epfl.ch/software-tools/blast/
- CPAchecker
  - http://cpachecker.sosy-lab.org/
- UFO/Whale
  - https://bitbucket.org/arieg/ufo
- Wolverine
- ... and many others

Further reading

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- T. Ball, V. Levin, and S.K. Rajamani. **A Decade of Software Model Checking with SLAM**. Communications of the ACM, 54(7), 2011