Static Analysis: Overview, Data-Flow

http://d3s.mff.cuni.cz

Department of Distributed and Dependable Systems

Pavel Parízek

CHARLES UNIVERSITY IN PRAGUE

faculty of mathematics and physics

Static analysis

- Purpose
  - Gather information about run-time behavior of programs without executing them

- Information
  - Does the variable x have a constant value?
  - Is the value of the variable x always positive?
  - May the pointer p be null at a code location?
  - What are possible values of the variable y?

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Static analysis: characteristics

- Target model of program behavior
  - some kind of ***Control Flow Graph*** **(CFG)**
- Provides **approximate** answers
  - Decision problems: yes / no / don't know
  - Collecting some values: superset / subset
- Information valid for all possible runs
- Summarizing different execution paths
  - branches of the `if-else` statement, loop iterations
- Does not know run-time values (inputs)

Comparison

Static analysis	Model checking
control-flow graph	program state space
summarizes information from different paths	reasons about execution paths independently
approximation	path-sensitivity
scalability	precision

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Static analysis in practice

- Optimizing compilers
  - Detect superfluous evaluations of the same expression
  - Detect unused local variables or dead code fragments

- Program verification
  - Search for possible runtime errors
    - Example: null pointer dereference, unsynchronized access
  - Constructing abstraction for model checking
    - Slicing: identify statements irrelevant for a given property

Approximation

Q: What important restrictions there are?

Restrictions

- Approximation must be safe
  - That precisely means “imprecise on the safe side”

- Target domain: **optimizing compilers**
  - Under-approximation
    - Optimization performed on the basis of analysis results must not violate semantics of a given program
  - Example: constant propagation
    - Sound analysis identifies a program variable as a constant only when it is really certain (100%)

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Restrictions

- Approximation must be safe
  - That precisely means “imprecise on the safe side”
- Target domain: **search for errors**
  - Over-approximation
    - Safe analysis reports all real errors and also some spurious errors (false positives)
  - Example: possible null dereferences
    - We want to know about all of them so we can add runtime checks (`if (v != null) ...`)

Basic concepts (theory and examples)

Running example

- Program

```
int factorial(int n) {
  int r;
  if (n == 0) r = 0;
  int f = 1;
  while (n > 0) {
    f = f * n;
    n = n - 1;
    if (n == 0) r = f;
  }
  return r;
}
```

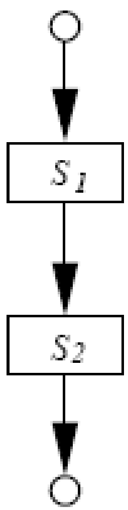
- Static analysis: **possibly uninitialized variables**

Control flow graph (CFG)

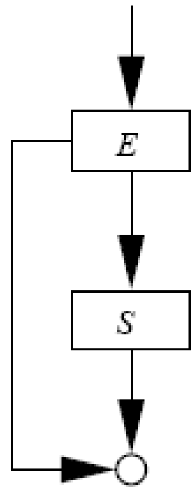
- Directed graph with labels
- Nodes: program points (statements)
- Edges: possible flow of control
  - *pred*(n) and *succ*(n) for each node n in a CFG
- Single point of entry
- Single point of exit

CFG: modeling control structures

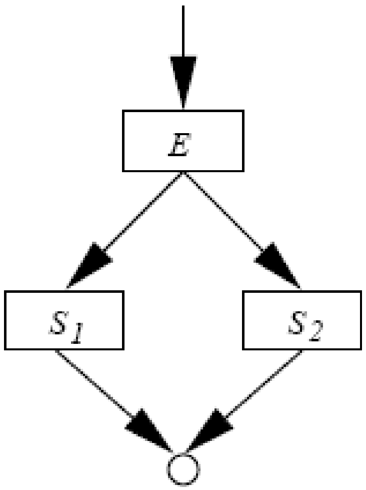
sequence
$S_1; S_2$



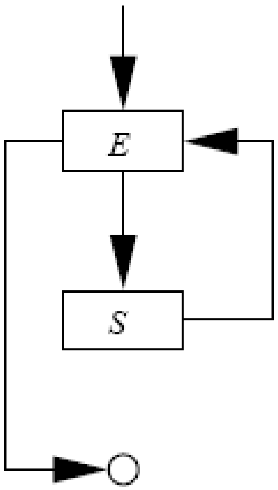
`if (E) {S}`



`if (E) {S1}`
`else {S2}`



`while (E) {S}`



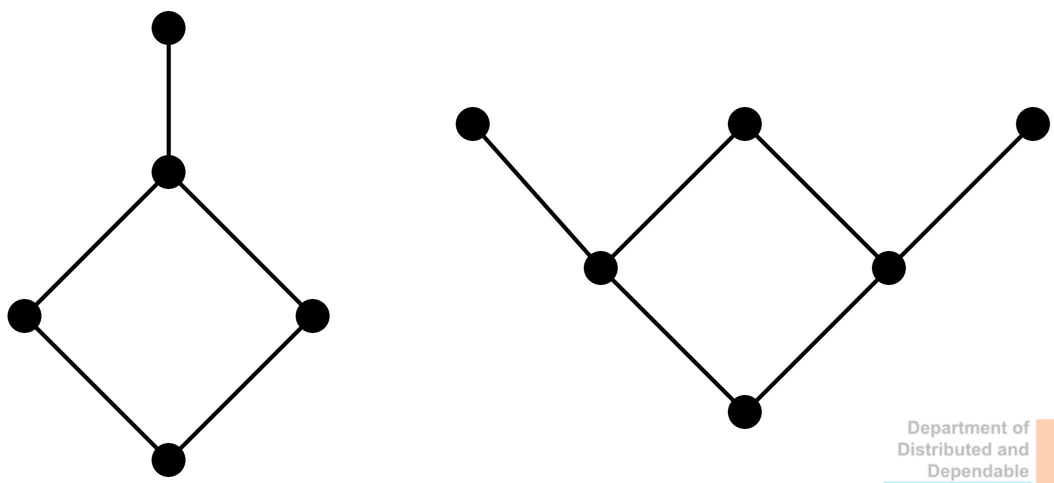
Analysis domain

- Set of possible values (facts)
- Finite lattice over the set

Partial order

- Mathematical structure $L = (S, \sqsubseteq)$
  - S is a set of values (e.g., analysis facts)
  - $\sqsubseteq$ is a binary relation (e.g., is-subset)
    - Reflexivity: $\forall x \in S : x \sqsubseteq x$
    - Transitivity: $\forall x,y,z \in S : x \sqsubseteq y \land y \sqsubseteq z \Rightarrow x \sqsubseteq z$
    - Anti-symmetry: $\forall x,y \in S : x \sqsubseteq y \land y \sqsubseteq x \Rightarrow x = y$

- Examples



Bounds

Lets have a partial order $L = (S, \sqsubseteq)$ and $X \subseteq S$

- Upper bound
  - $y \in S$ is an upper bound for X, i.e. $X \sqsubseteq y$, if $\forall x \in X : x \sqsubseteq y$
- Lower bound
  - $y \in S$ is a lower bound for X, i.e. $y \sqsubseteq X$, if $\forall x \in X : y \sqsubseteq x$

- Least upper bound of X, denoted as $\sqcup X$
  - $X \sqsubseteq \sqcup X \;\; \land \;\; \forall y \in S : X \sqsubseteq y \Rightarrow \sqcup X \sqsubseteq y$
- Greatest lower bound of X, denoted as $\sqcap X$
  - $\sqcap X \sqsubseteq X \;\; \land \;\; \forall y \in S : y \sqsubseteq X \Rightarrow y \sqsubseteq \sqcap X$

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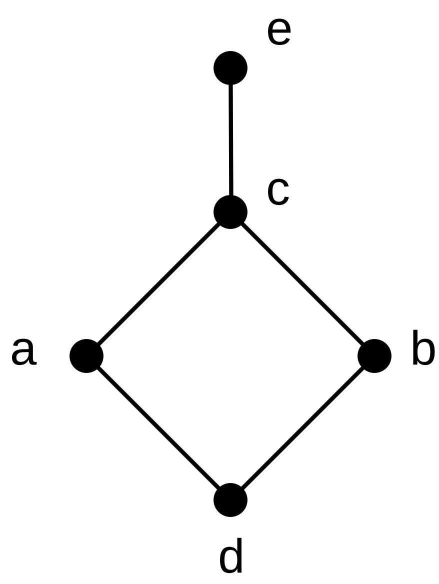
Bounds: example 1

Lets have a partial order $L = (S, \sqsubseteq)$ and the set $S = \{a, b, c, d, e\}$

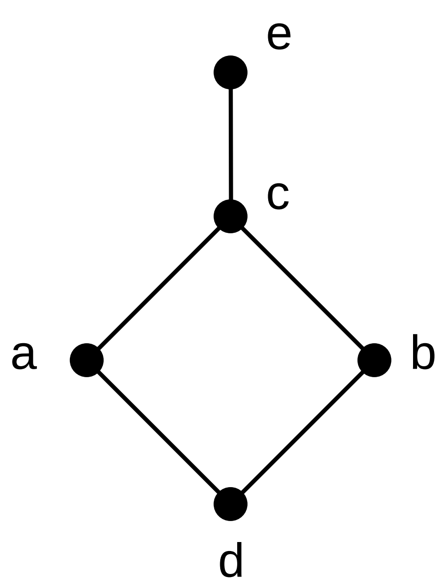
The upper bounds of $X = \{a, b\}$ are the elements $\{c, e\}$



Bounds: example 2

Lets have a partial order $L = (S, \sqsubseteq)$ and the set $S = \{a, b, c, d, e\}$

The greatest lower bound of $X = \{b, e\}$ is the element b



Lattice

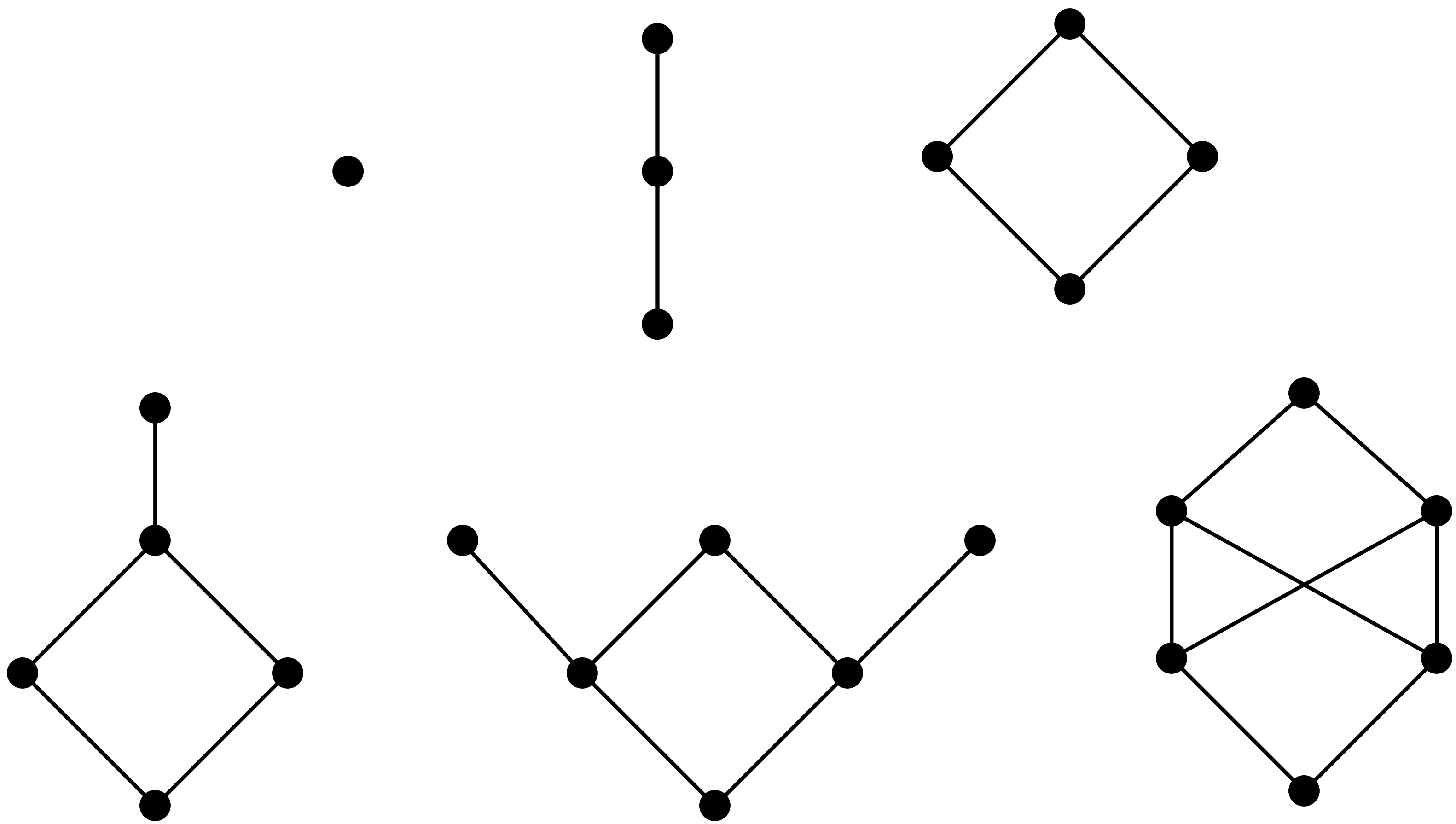
- Partial order $L = (S, \sqsubseteq)$ such that
  - $\sqcup X$ and $\sqcap X$ exist for $\forall X \subseteq S$
  - Unique greatest element $\top = \sqcup S = \sqcap \emptyset$
  - Unique least element $\bot = \sqcap S = \sqcup \emptyset$

- Height of a lattice
  - Length of the longest path from $\bot$ to $\top$

Finite lattice

- Partial order $L = (S, \sqsubseteq)$ such that
  - $\forall x,y \in S$ there is
    - Least upper bound $x \sqcup y$
    - Greatest lower bound $x \sqcap y$

Lattice: examples

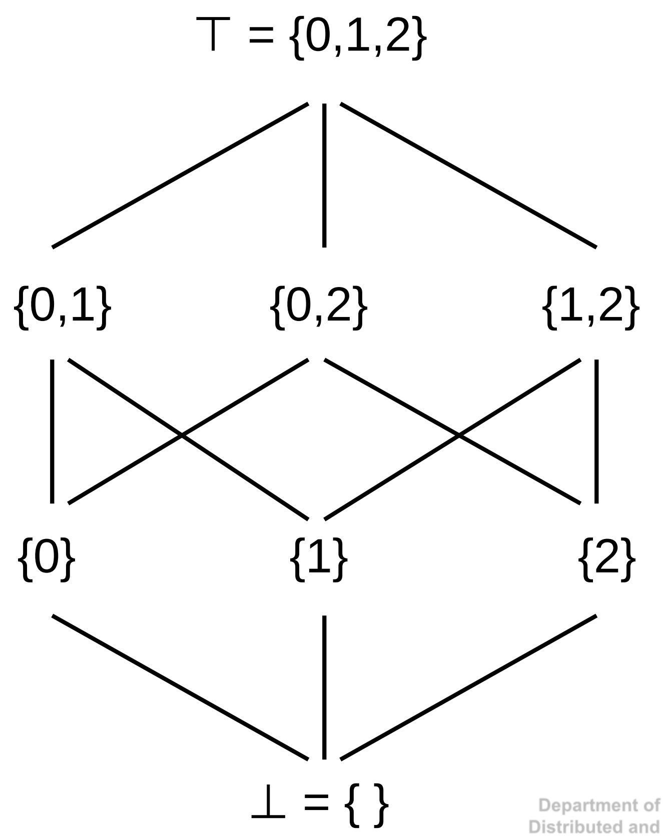


Using finite lattices in static analysis

- Lattice $L = (S, \sqsubseteq)$
  - Set S of analysis facts (units of information)
  - Relation $\sqsubseteq$ defines an ordering with respect to precision of the abstraction
    - $x \sqsubseteq y \Rightarrow x$ is more precise than y
    - $x \sqsubseteq y \Rightarrow y$ approximates x
  - Example
    - Sign abstraction: x = { POS }, y = { POS, ZERO }

How to construct lattices

- Finite set R induces a lattice $(2^R, \sqsubseteq)$
  - $\bot = \sqcup\varnothing$
    - No information available
  - $\top = R$
    - Any possible value
  - $x \sqcup y = x \cup y$
  - $x \sqcap y = x \cap y$
  - Height $|R|$
- Example
  - Set $R = \{0, 1, 2\}$
  - Height = 3



Running example

- Program

```
int factorial(int n) {
  int r;
  if (n == 0) r = 0;
  int f = 1;
  while (n > 0) {
    f = f * n;
    n = n - 1;
    if (n == 0) r = f;
  }
  return r;
}
```

- Static analysis: possibly uninitialized variables

Encoding program statements

- Data for each node in the CFG
  - IN: valid before the program statement
  - OUT: valid after the program statement
- Merge operator $\sqcup$
  - CFG nodes with multiple predecessors
  - Typical approach: union or intersection
- Transfer functions

Transfer functions

- For each node in CFG (statement), we must define a transfer function

  OUT = (IN \ kill) U gen

- Examples
  - Statement `int r;`

    kill = {}, gen = { r }
  - Statement `r = f;`

    kill = { r }, gen = {}

Monotone functions

- Function $f : S \rightarrow S$ is monotone if
  - $\forall x, y \in S : x \sqsubseteq y \Rightarrow f(x) \sqsubseteq f(y)$
- Examples
  - Constant functions
  - Operators $\sqcap$ and $\sqcup$
  - Their compositions

Computing static analysis

- Input
  - Control flow graph of the given program
  - Initial value for each CFG node ($\perp$ or $\emptyset$)
    - Value is the set of known analysis facts (information)
  - Merge operator defined as the set union
  - Transfer functions F_i for each node in CFG
- Approach: **compute fixed points**
  - Information associated with the CFG nodes

Duality

$(S, \sqsubseteq)$ is a lattice $\Leftrightarrow$ $(S, \sqsupseteq)$ is a lattice

$\bigsqcup_{(S, \sqsubseteq)} = \bigsqcap_{(S, \sqsupseteq)}$ $\top_{(S, \sqsubseteq)} = \bot_{(S, \sqsupseteq)}$

$\bigsqcap_{(S, \sqsubseteq)} = \bigsqcup_{(S, \sqsupseteq)}$ $\bot_{(S, \sqsubseteq)} = \top_{(S, \sqsupseteq)}$

- We focus just on $\sqsubseteq$ and initial values $\bot$

Computing fixed points

- Motto: "*walk up the lattice starting at $\perp$, until you reach a fixed point*"
  - In the worst case, $\top$ is the fixed point (if exists)

- Three algorithms
  - Naive (brute force)
  - Chaotic iteration
  - **Worklist algorithm**

Worklist algorithm

```
u₁ = ⊥; ..., uₙ = ⊥;
q = [1, ..., n];
while (q ≠ []) {
  i = head(q);
  v_IN = merge(pred(i));
  v_OUT = F_i(v_IN);
  q = tail(q);
  if (v_OUT ≠ u_i) {
    append(q, succ(i));
    u_i = v_OUT;
  }
}
```

Classification

Static analysis categories

- Data-flow analysis

- Call graph construction
- Pointer analysis (aliasing)
- Escape analysis (threads)
- Side effect analysis

Data-flow analysis

- Available expressions
- Reaching definitions
- Live variables (values)

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Available expressions

```
var x,y,a,b;
y = a - b;
while (y < a + b) {
  a = a - 1;
  x = a + b;
}
```

```
var x,y,a,b,t;
y = a - b;
t = a + b;
while (y < t) {
  a = a - 1;
  t = a + b;
  x = t;
}
```

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Direction

- Forward analysis
  - Computes information about the past behavior
  - Starts at the entry node (CFG) and goes forward

- Backward analysis
  - Computes information about the future behavior
  - Starts at the exit CFG node and moves backwards

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Approximation level

- May analysis
  - Computes information that may be true (over-approximation)
    - Information for P that is true at least for one path coming into P
  - Merge operator: set union

- Must analysis
  - Computes information that must be true (under-approximation)
    - Information for P that is true for all execution paths coming into P
  - Merge operator: set intersection

Flow sensitivity

- Flow-sensitive analysis
  - Considers the program’s control flow (CFG) and the order of individual statements
  - Example: available expressions
- Flow-insensitive analysis
  - Program seen as an unordered collection of statements
  - Results are valid for any order of program statements
    - *S1* ; *S2* versus *S2* ; *S1*
  - Example: type analysis (inference)

Scope

- Intra-procedural
  - Every single procedure analyzed separately
  - Maximally pessimistic assumptions about side effects of procedure calls
- Inter-procedural
  - Whole program analyzed together
  - Sometimes without libraries (huge)

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Context sensitivity

- Context-sensitive analysis
  - Call site: source code location for the call
  - Call stack: procedure calls and returns
  - Receiver objects for method calls (“`this`”)
  - Analysis results for the method M depend on the specific caller of M
- Context-insensitive analysis
  - Same analysis results for every call site of M

Tools

- WALA
  - Java, JavaScript, JVM (bytecode)
  - http://wala.sourceforge.net/
  - https://wala.github.io/
- Soot
  - Java, JVM-based languages (bytecode)
  - http://sable.github.io/soot/
- CIL
  - Only for programs written in C
  - http://www.cs.berkeley.edu/~necula/cil/
  - https://github.com/cil-project/cil
- LLVM
  - C, C++, Objective-C
  - Clang static analyzer
  - http://llvm.org/

Further reading

- M. Schwartzbach. **Lecture Notes on Static Analysis**. Department of CS, Aarhus University
- F. Nielson, H. R. Nielson, and Chris Hankin. **Principles of Program Analysis**. Springer, 2005

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