Decision Procedures and Verification

Seminar 3

1. (1 point) Simulate GSAT on the following CNF formula given as a list of clauses:
   $(x_5 \vee \neg x_1 \vee x_3)$
   $(\neg x_3 \vee x_4)$
   $(\neg x_5 \vee x_1)$
   $(x_6 \vee x_1)$
   $(\neg x_1 \vee x_2)$
   $(\neg x_3 \vee \neg x_4)$
   $(\neg x_5 \vee \neg x_6)$
   $(x_3 \vee \neg x_4)$
   $(x_3 \vee x_5)$
   $(x_2 \vee x_4 \vee \neg x_5)$
   $(x_4 \vee \neg x_5 \vee \neg x_6)$
   $(x_6 \vee \neg x_1)$
   $(x_3 \vee \neg x_5)$
   $(x_5 \vee x_1)$
   $(\neg x_6 \vee \neg x_3 \vee x_2)$
   $(x_1 \vee \neg x_2 \vee \neg x_4)$
   $(\neg x_2 \vee \neg x_4 \vee x_6)$

2. (0.5 points) Solve the previous formula without any decisions. Use only preprocessing techniques and unit propagation.

3. (1 point) (blocked clauses) Let φ be a CNF formula, let $c \in \varphi$ be a clause such that $l \in c$ where l is a literal, and let $\varphi_{\neg l} \subseteq \varphi$ be the subset of φ's clauses that contain $\neg l$. We say that c is *blocked* by l if the resolution of c with any clause in $\varphi_{\neg l}$ using $var(l)$ as the pivot is a tautology. For example, if $c = l \vee x \vee y$ and $\varphi_{\neg l}$ has a single clause $c' = (\neg l \vee \neg x \vee z)$, then resolving c and c' on l results in $x \vee \neg x \vee y \vee z$, which is a tautology, and hence c is blocked by l. Prove that φ is equisatisfiable to $\varphi \setminus c$, i.e., blocked clauses can be removed from φ without affecting its satisfiability.

4. (0.5 points) Construct a factor graph for a CNF formula $\varphi = (x_1 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_4) \wedge (\neg x_3 \vee x_5) \wedge (\neg x_3 \vee \neg x_4 \vee x_5) \wedge (\neg x_2 \vee x_4 \vee x_6) \wedge (x_5)$

5. (1 point) Consider a factor graph from the following image where edges are labeled with warnings. $x_1, \ldots, x_8$ are variables and $c_1, \ldots, c_9$ are clauses. Dotted edges denote negative occurrences variables in clauses while solid edges represent positive occurrences. Simulate warning propagation.