Decision Procedures and Verification

Seminar 4

1. (1 point) Decide the following QBF: $\psi = \forall y_5 \exists x_1 \forall y_2 \exists x_3, x_4 \; (\neg y_5 \vee x_4) \wedge (y_5 \vee \neg x_4) \wedge (x_1 \vee y_2 \vee \neg x_4) \wedge (\neg x_1 \vee x_3 \vee \neg x_4) \wedge (\neg y_2 \vee \neg x_3)$.
2. (1 point) Construct a BDD for $\neg(x_1 \vee (\neg x_2 \vee \neg x_3))$
   1. Use reduction from binary decision tree
   2. Use inductive construction
3. (1 point) Let $f : \{0,1\}^n \to Z$ be a non-Boolean function mapping a Boolean vector to an integer. Let $\{I_1, I_2, \dots, I_N\}$ where $N \leq 2^n$ be the set of possible values of f. The function partitions the space $\{0,1\}^n$ of Boolean vectors to N sets $S_1, S_2, \dots, S_N$ such that for $i \in \{1, 2, \dots, N\}$ $S_i = \{\vec{x} \in \{0,1\}^n | f(\vec{x}) = I_i$. Suggest a multi terminal binary decision diagram (MTBDD) with $I_1, I_2, \dots, I_N$ as its terminal nodes that represents f. Build MTBDD for $f(x, y) = 2x + 2y$.
4. (1 point) Prove the correctness of universal reduction in QBF solving.
5. (1 point) Prove the correctness of pure literal propagation in QBF solving.